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Particle Physics 2: Quantum Chromodynamics

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1 Historical Motivation for Quarks and Color

In this second part of the Particle Physics 2 course, we introduce students to Quantum Chromodynamics (QCD), the quantum field theory (QFT) describing the strong nuclear force. The QCD part of the course is composed by 7 lectures of 1.45h each, each of them accompanied by a 1.45h tutorial session with hands-on exercises and numerical simulations, as well as by guest lectures and by a paper reading exercise. The contents to be discussed are:

- Historical introduction and the QCD Lagrangian.
- Infrared and collinear divergences: QCD in electron-positron collisions.
- Deep-inelastic scattering: QCD in lepton-proton collisions.
- Higgs production: QCD in proton-proton collisions.
- Parton showers, Monte Carlo generators, and jet reconstruction in QCD.

We begin in this lecture by first providing a brief recap of the historical developments which lead to the formulation of QCD.

Learning Goals of the Lecture

- Identify main characteristics of the quark degrees of freedom.
- Establish the consequences of color in the hadron spectrum and in high-energy processes.
- Describe the main experimental analyses confirming the existence of quarks and the associated color degree of freedom.

To begin, let us discuss the main pieces of evidence that historically were crucial to convince physicists of first of all the existence of a **new quantum number (color)** and second of the real existence of quarks as components of hadrons, the two basic ingredients of QCD.

1.1 SU(3) symmetry and evidence for color

Historically, the color quantum number was introduced to explain some puzzling features of the hadron spectrum. In the 60s a large number of strongly interacting particles had been discovered: pions, kaons, baryons, etc, and the question was whether they were fundamental or composite by more fundamental degrees of freedom. Gell-Mann and Zweig introduced the **quark model** which allowed to organize hadrons in terms of quarks, hypothetical particles with following properties:

- Quarks are fermions with spin $s = 1/2$.
- Quarks they have fractional electric charge: $Q_q = +2/3$ or $1/3$.
- Quarks exists in three flavors, called *up*, *down* and *strange*, which lead to the observed hadron spectrum.

This worked fine to describe many of the observed hadrons. For example, the pions could be explained in the quark model as follows

$$|\pi^+\rangle = |u\bar{d}\rangle\,, \qquad |\pi^-\rangle = |d\bar{u}\rangle\,. \tag{1.1}$$

However, other strongly interacting particles offered more challenges. In particular, we have the Δ^{++}, a baryon which in the parton model has assigned the following quark decomposition

$$|\Delta^{++}\rangle = |uuu\rangle\,, \tag{1.2}$$

which is forbidden by quantum mechanics, since this baryon is a **fermion** however its wave function seems to be **symmetric** upon the exchange of any pair of quarks.

Evidence for color

To make sense of particles like Δ^{++}, composed by three up quarks with the same spin, a new physical property, without classical analog, needs be introduced:

- Quark have a new internal degree of freedom, color, which can take three possible values ($N_C = 3$).
- Color is confined in Nature, so all observed hadrons should be **color-singlet**. In other words, color cannot exist in isolation, at least under standard conditions.

With this extension, we can avoid Pauli's exclusion principle and make sense of the Δ^{++} particle and similar baryons.

In Fig. 1.1 we summarize the properties of known quarks, and illustrate the baryon decuplet with its various quantum numbers. In the latter, states are classified by their strangeness content and the electric charge.

Discussion #1.1

Why only three families of quarks exists? Could there be more? Could they be less? What are the implications? Do we actually need the 2nd and 3rd generation?

While the quark model provided a handy mechanism to classify the experimentally observed hadrons, it was not clear whether quarks had a physics, rather than mathematical, existence. Subsequent experimental studies were required to further assert the physical existence of quarks as constituents of hadrons.

Electric Charge= $\frac{2}{3}e$ $m =$	up few MeV	charm ≈ 1.5 GeV	top $\approx$170 GeV
Electric Charge= $-\frac{1}{3}e$ $m =$	down few MeV	strange few hundred MeV	bottom ≈ 5 GeV

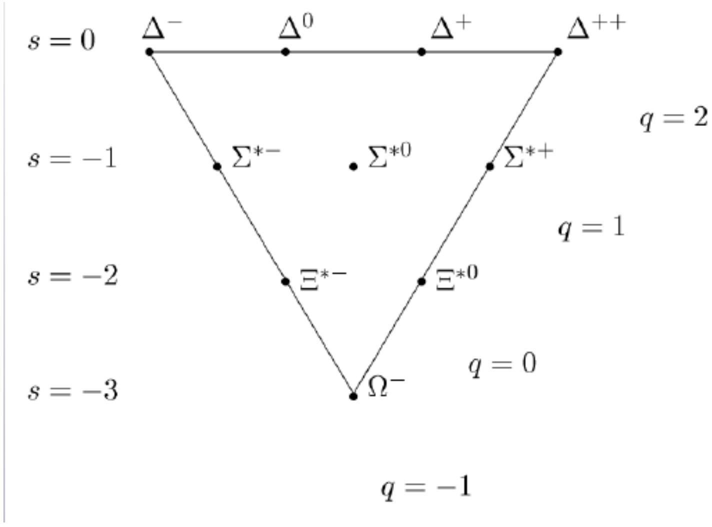

Figure 1.1. Upper table: properties of quarks. Lower plot: the baryon decuplet with its various quantum numbers.

1.2 Deep-inelastic scattering and the R ratio: evidence for quarks

QCD and the color quantum number appeared to be a nice mathematical approach to organize the structure of all known hadrons. The evidence for their existence of quarks as real particles, as constituents of hadrons, was provided only in the late 60s and early 70s by a series of experiments called Deep-Inelastic Scattering (DIS), which is one of the cornerstones of QCD.

Quarks in deep inelastic scattering. DIS denotes the inelastic scattering of a highly energetic electron or muon off a static proton target, as indicated in Fig. 1.2:

$$e^-(k) + p(P) \to e^-(k') + X(p_h)\,, \tag{1.3}$$

where in general the proton will be destroyed by the collision (hence the name *inelastic*). The four momentum transfer between the lepton and the proton is

$$q \equiv k - k'\,. \tag{1.4}$$

The kinematics of the deep-inelastic scattering process are completely specified by the following variables

$$x_{\rm Bj} \equiv \frac{Q^2}{2\,P\cdot q}, \quad Q^2 \equiv -q^2\,, \quad y \equiv \frac{q\cdot P}{k\cdot P}\,. \tag{1.5}$$

As we will see soon, the so-called Bjorken-x variable is related to the momentum fraction carried by quarks. While Eq. (1.3) is valid in any reference frame, since in the early experiments the proton target was at rest,

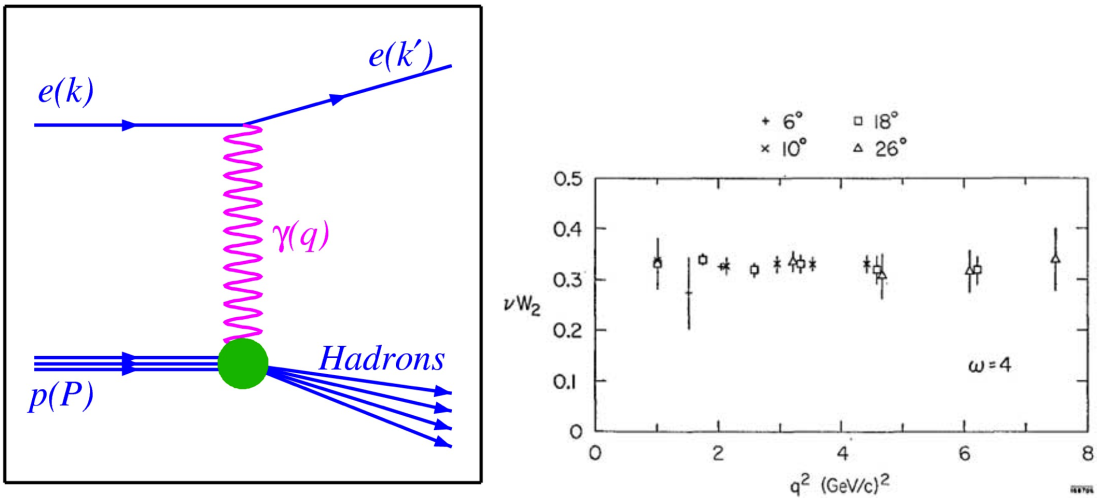

Figure 1.2. Left plot: the deep-inelastic scattering (DIS) process. Right plot: the original SLAC DIS structure function measurements, as a function of the four-momentum transfer between the electron and the proton.

it is advisable to work in the **target rest frame**:

$$k^\mu = (E_\ell, 0, 0, E_\ell)\,, \qquad P^\mu = (m_p, 0, 0, 0)\,, \tag{1.6}$$

$$k'^\mu = (E'_\ell, \vec{p}_\ell{}')\,, \qquad p_h^\mu = (E_h, \vec{p}_h)\,, \tag{1.7}$$

where p_h is the momentum of the hadronic final state (proton remnant), and we neglect the incoming lepton mass. For example, in this rest frame the **inelasticity** y is given by

$$y = \frac{q\cdot P}{k\cdot P} = 1 - \frac{k'\cdot P}{k\cdot P} = 1 - \frac{E'_\ell}{E_\ell}\,, \tag{1.8}$$

that is, at the fraction of energy of the initial-state charged lepton which is transferred to the hadronic final state ($y = 1$ corresponds to $E'_\ell = 0$ that is all energy is transferred to the proton system).

The center-of-mass energy of the proton-virtual photon collision will be, in a general reference frame

$$W^2 \equiv (P+q)^2 = P^2 + 2P\cdot q + q^2 = m_p^2 + Q^2\frac{1-x}{x}\,, \tag{1.9}$$

To differentiate from *elastic scattering*, the condition must be that $W^2 \gg m_p^2$: the invariant mass of the proton-virtual photon collision to be larger than the proton rest mass. This translates to $Q^2(1-x) \gg m_p^2$, else the proton would not be destroyed. The value $x = 1$ is known as the **elastic limit** of DIS.

From the Rutherford scattering experiments to SLAC

In the early 1900, the Rutherford scattering experiment led to the discovery of the atomic nucleus, by showing that the deflection of α-particles impinging in a thin gold plate was described by scattering off point particles rather than a more or less homogeneous distribution of positive electric charge. Likewise, the existence of quasi-free, point-like objects in the proton with fractional electric charge can be inferred from the angular deflection of leptons after the scattering with protons in the target. If the proton is composed by point like objects the scattering distribution will be very different as compared to an homogeneous distribution of charge within the proton.

A powerful property of DIS is that the complete kinematics of the process are fully specified by measuring the four-momenta of the outgoing lepton, if the energy of the incoming lepton is known. In the parton model, which we will discuss in more detail later in the lectures, **scaling** is the statement that, after removing the trivial kinematic factors, the cross-section does not depend on the four-momentum transfer in the collision, Q^2, or in other words, that the scattering centers in the proton are point-like objects, rather than some smooth charge distribution which would lead to a **form factor instead**. In the parton model of QCD, the measured SLAC cross-sections can be written as

$$\nu W_2(Q^2,\nu) = \frac{Q^2}{m_p x} W_2(Q^2,x) \equiv F_2(x,Q^2) = x \sum_i e_q^2 q_i(x) \, . \quad (1.10)$$

The cross-section for DIS structure functions is proportional to the **momentum fraction carried by the quarks inside the proton** $q_i(x)$, weighted by the quark charge. These momentum distributions, known as PDFs, indicate the fraction of the proton's momentum being carried by each individual quark flavor. The scaling of the original SLAC data can be seen in Fig. 1.2. Perturbative QCD induces logarithmic corrections in Q^2 in Eq. (1.10), which have been observed by more recent DIS experiments and that we will see during these lectures.

Discussion #1.2

Can we probe, via the same DIS process, the existence of **other possible components in the proton** which are electrically neutral and do not modify the proton quantum numbers? Which could be the associated Feynman diagram?

Discussion #1.3

Can you think of a qualitative argument which justifies why a point-like composition of the proton should have associated a **flat Q^2 dependence**, once kinematic factors have been removed?

Quarks in e^+e^- annihilation. Another crucial piece of evidence for both color and quarks was provided by the measurement of the ratio of the cross-section $\sigma(e^+e^- \to \text{hadrons})$ over $\sigma(e^+e^- \to \mu^+\mu^-)$, know as the R-ratio as a function of $\sqrt{s}$, the center of mass energy of the electron-positron collision. This ratio is defined as:

$$R(\sqrt{s}) \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \, , \quad (1.11)$$

and the associated Feynman diagrams are shown in Fig. 1.3. The ratio is useful since we can test the existence of quarks and of the color quantum number as follows:

QCD in e^+e^- annihilation.

- If we produce a quark-antiquark pair rather than a muon-antimuon pair, the Feynman diagram receives a contribution of a factor Q_q where $Q_q e$ is the quark charge.
- If there are N_c colors, then a quark-anti-quark pair can be produced in N_c different ways (recall that the intermediate photon is a color singlet).
- A quark-antiquark pair can only be produced if $\sqrt{s} = 2m_q$ where m_q is the quark mass. By checking that $R(\sqrt{s})$ increases as we cross a heavy quark threshold, this provides additional evidence of their existence.

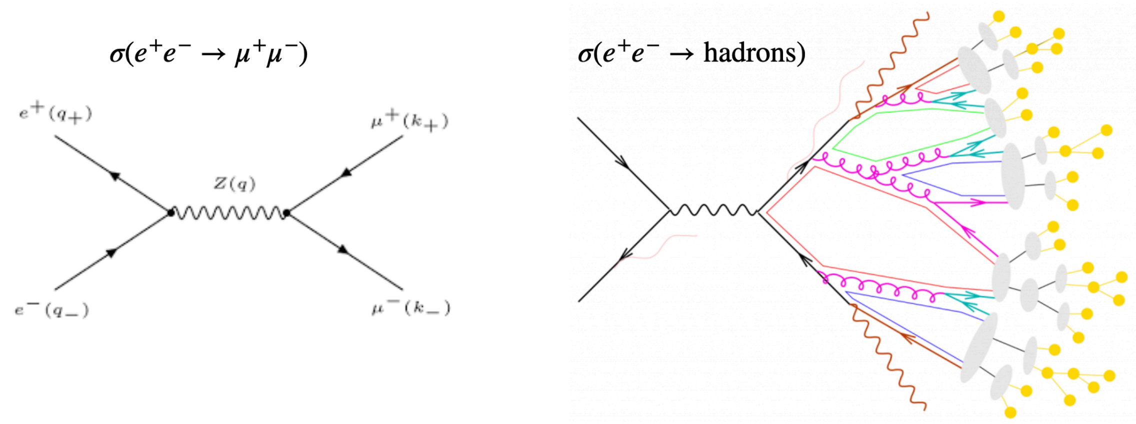

Figure 1.3. Feynman diagrams for the production of muon/antimuon pairs (left) and of hadrons (right panel) in electron-positron annihilation. In the latter, the yellow blobs indicate the final state stable hadrons.

Taking into account these considerations, we expect that this observable behaves as follows:

$$R(\sqrt{s}) \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = N_c \sum_{q=1}^{N_q} Q_q^2 \theta\left(\sqrt{s} - 2m_q\right) \, , \tag{1.12}$$

where the sum runs over all N_q quarks predicted by QCD. Only quarks light enough to be produced at any given energy $\sqrt{s}$ are included in the sum. Indeed, the dependence of the R ratio with $\sqrt{s}$ is also useful to determine the mass of the heavy quarks: only for $\sqrt{s} \geq 2m_h$ can heavy quarks be produced in e^+e^- annihilation and thus contribute to the R ratio.

In Fig. 1.4 we show the R ratio as a function of $\sqrt{s}$. In addition to the resonances, for the continuum cross-section we see that the overall value is in good agreement with the QCD expectations based on the parton model, and indeed we see the change of value when crossing heavy quark thresholds, in particular charm and bottom thresholds. Above the bottom threshold the value of the R-ratio stabilizes to

$$R(\sqrt{s} \geq 2m_b) \equiv R_5 = \frac{11}{3} = 3\left[2\left(\frac{2}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2\right] \, , \tag{1.13}$$

As can be seen from Fig. 1.4, the data strongly disagrees with the possibility that quarks carry no color quantum number.

Discussion #1.4

Why the formation of hadronic resonances lead to a large enhancement of the $\sigma(e^+e^- \to \text{hadrons})$ cross-section? Can we draw the corresponding Feynman diagrams? What this is telling us about the strong interaction?

The answer is that Feynman diagrams in Fig. 1.3 are not appropriate if we create a hadronic resonance like the Υ. In this case we have

$$e^+ + e^- \to \gamma^* \to \Upsilon \to \text{hadrons} \, , \tag{1.14}$$

that is, the Υ resonance couples directly to the virtual photon and the cross-sections receives a Breit-Wigner

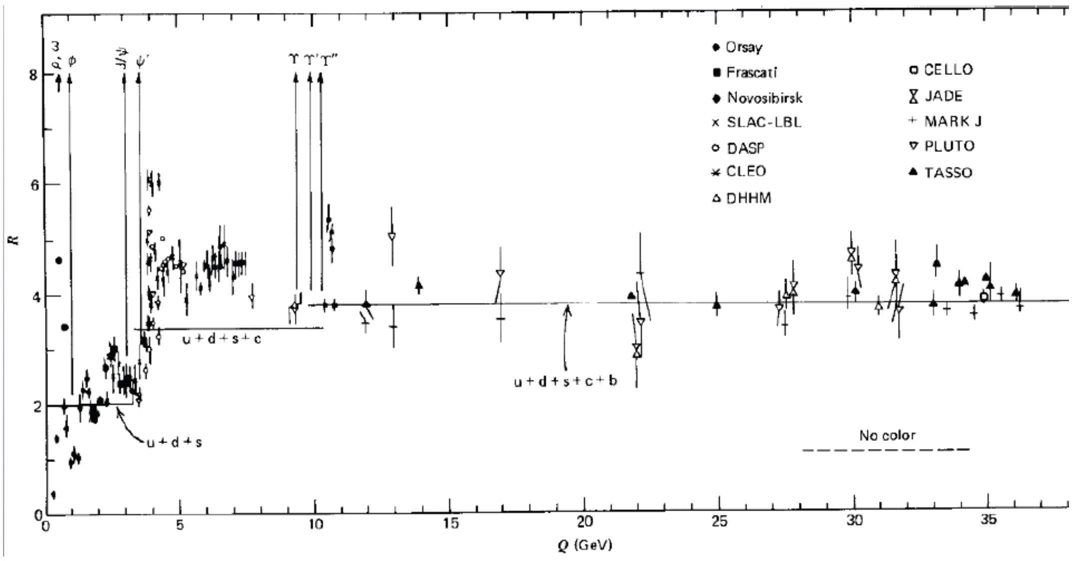

Figure 1.4. The ratio R of hadron production in electron-positron annihilation over muon-antimuon pairs, Eq. (1.12) as a function of $\sqrt{s}$, the center-of-mass energy of the collision. The dashed horizontal line indicates the theoretical prediction should the color quantum number not exist.

enhancement

$$\sigma(\sqrt{s}) \propto \frac{1}{(s - m_\Upsilon^2)^2 + m_\Upsilon^2 \Gamma_\Upsilon^2} \, . \tag{1.15}$$

The fact that near the resonance mass this process becomes much larger than those of Fig. 1.3, which are mediated by QED, tells us that the **strong interaction is much more intense than the electromagnetic one**.

Discussion #1.5

Has the top quark ever been produced in e^+e^- annihilation? Why not?