The Standard Model Effective Theory: towards a pedagogical primer

Juan Rojo[1*],

1 VU Amsterdam and Nikhef Theory Group
* j.rojo@vu.nl

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Abstract

There notes provide some supporting material for the lectures of the course "The Standard Model as an Effective Field Theory", which was part of the 2020 DRSTP School of Theoretical High-Energy Physics (Dalfsen, The Netherlands, February 2020). Reflecting the intrinsic optimism (or perhaps better naivete) of the lecturer, these notes might eventually evolve into a full-fledged primer on the SMEFT (hence the SciPost template) but that day still lies far in the future.

The current version of these notes is February 4, 2020. These notes are still work in progress and certified to be affected by typos, inconsistent notation, and also blatant mistakes (not to mention a complete lack of bibliography) so use at your own risk.

Contents

1 Introduction

In these notes we provide some introduction to the Standard Model Effective Theory, the so-called SMEFT. These notes are based on the lectures given at the DRSTP School of Theoretical High-Energy Physics, which took place in Dalfsen (The Netherlands) between the 27th of January and the 7th of February 2020. The full program of the school can be found here:

https://web.science.uu.nl/drstp/Postgr.courses/THEP/2020/info.html

This school was aimed to the Ph. D. students belonging to the Dutch Research School of Theoretical Physics (DRSTP). Given than the participating students have different levels of background preparation concerning particle physics phenomenology, these lectures try to avoid technicalities as much as possible and focus on the more relevant qualitative features of the SMEFT formalism and its phenomenological implications.

These lecture notes have been compiled with the help of material contained in a number of reviews, articles, and lecture materials on the general topic of Effective Field Theories and on the SMEFT in particular. An incomplete list of references, that the interested student might consult to deepen her knowledge of the topic, are the following:

- I. Brivio and M. Trott [1], "The Standard Model as an Effective Field Theory," Phys. Rept. **793**, 1 (2019) [arXiv:1706.08945 [hep-ph]].
- T. Cohen [2], "As Scales Become Separated: Lectures on Effective Field Theory," PoS TASI **2018**, 011 (2019) [arXiv:1903.03622 [hep-ph]].
- A. V. Manohar, "Introduction to Effective Field Theories," arXiv:1804.05863 [hep-ph].

Other resources that have been consulted when preparing these lectures include the lectures by Francesco Riva at the first EWSB Spring School 2018,

https://indico.cern.ch/event/673580/timetable/

as well as the MIT on-line EFT course materials, publicly available from

https://www.edx.org/es/course/effective-field-theory

I am also grateful to a number of colleagues that have provided useful references and suggestions during the preparation of these lectures: Ilaria Brivio, Fabio Maltoni, Francesco Riva Mike Trott, and Eleni Vryodinou.

Lecture plan. The course is organised into four 1.5h lectures and two 1.5h hands-on tutorials. First of all, I start in the first part of the course, Sect. 2, by introducing the main ideas underlying effective field theories and providing some pedagogical examples. Then I move to present the SMEFT and its main properties in Sect. 3, including explicit matching examples and a first discussion of the kind of effects that the presence of the SMEFT higher-dimensional operators induce. The third and final part of this lectures focuses on phenomenological analyses of the SMEFT, Sect. 4, at the LHC and other colliders, covering also a discussion of the connection between different sectors (such as Higgs physics and flavour observables) and of the fitting methods that are used to extract bounds on the Wilson coefficients. We finally summarise a number of more advanced topics that are suggested for future study in Sect. 5. Some of the solutions of the proposed tutorial exercises are provided in Appendix A.

2 Basic concepts in Effective Field Theories

In this first lecture we introduce the basic ideas underlying effective field theories (EFT), and provide some examples that illustrates the main conceptual aspects that will appear in these lectures.

2.1 Separation of scales

The basic idea underlying the concept of effective theories is that in a wide range (but not all) of situations the physics that determine the behaviour of a system a given length or energy scale is decoupled from the physics that determines the behaviour of the same system at very different scales.

Some examples of this idea of separation of scales include:

- We don't need to know that Quantum Chromodynamics is the correct theory of the strong nuclear force (or even to know the existence of the strong nuclear force itself) to compute with high precision the electronic transitions in atoms.

  This property is a consequence of the fact that electronic transitions in atoms take place at length scales of $r_{\rm atom} \simeq 10^{-10}$ m, while the effects of the strong force arise only at much smaller scales $r_{\rm nucl} \simeq 10^{-15}$ m.

- We can compute with good accuracy the properties of the β radioactive decay of the neutron, $n \to p + e^+ + \nu_e$ without knowing about the existence of the W, Z bosons, or even about the quark substructure of hadrons for that matter.

- We can predict the motion of planets and solar eclipses with extremely high precision using only classical mechanics, which is only an approximation to the full theory, Special Relativity, valid when $v \ll c$.

In summary, the principle of separation of scales can be described as follows:

Principle of separation of scales

The physics that take place at a given distance or energy scale should be describable without the need of reference to the physics that take place at widely separated distance or length scales.

An Effective Field Theory (EFT) is one of the possible realizations of this concept, valid in Quantum Field Theories (QFT) that involve widely separated mass or energy scales. For example, consider a QFT composed by two scalar fields, a *light* field ϕ with mass m_ϕ and a *heavy* field ψ with mass m_ψ, so that $m_\phi \ll m_\psi$. Depending on the energy involved in the process, for example in a scattering amplitude, we will have two different regimes as illustrated in Fig. 2.1.

- For $E \gtrsim m_\psi$, both fields are dynamical degrees of freedom, and in particular appear in the initial and final states of scattering amplitudes. We denote this as the *full theory* or also as the *UV-complete theory.*

- For $E \ll m_\psi$ the heavy field is not dynamical, and in particular it cannot be kinematically produced in the final state of a scattering process. We say that this field can then be *integrated out* of the theory, to reflect that at low energies the only dynamical degree of freedom is the light field ϕ. We call the theory where ψ has been integrated out as the *effective theory.*

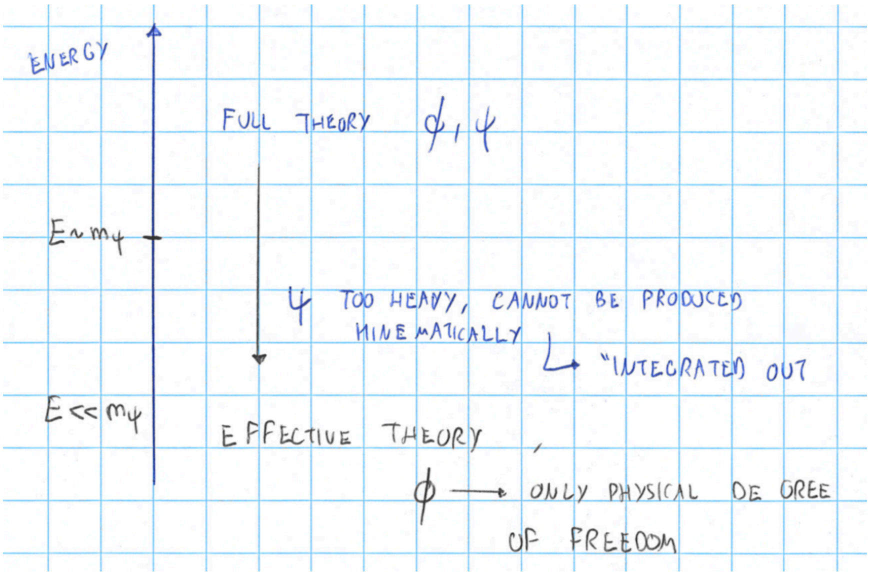


Figure 2.1: Separation of scales in a QFT composed by two scalar fields with a wide hierarchy of masses among them.

Note that the full and effective theory are not different theories: they are alternative formulations of the same physical dynamics, valid in a restricted region of energies. The main advantage of the EFT framework is that expression our theory in terms of the physical degrees of freedom relevant at low energy one can in general significantly simply calculations. We also note that in general one does not need to know the full theory to construct its effective version: as we will discuss, the EFT can be constructed based on general considerations without the requirement to refer to a full theory.

Before continuing with the formulation of EFT in quantum field theory, let us take back one step and present some examples of physical systems where the separation of scales simplifies the interpretation of the underlying dynamics.

2.2 The electrostatic dipole in 1D

Consider a one-dimensional system composed by two charges of same magnitude and opposite sign separated at a distance a. If the first charge is put at $x = 0$, the resulting potential is

$$V_{\rm full}(x) = Q\left(\frac{1}{|x|} - \frac{1}{|x-a|}\right) \,. \tag{2.1}$$

We want to evaluate this potential for $x = b > a$. In the "full theory" we will have

$$V_{\rm full}(x=b) = Q\left(\frac{1}{b} - \frac{1}{b-a}\right) \,. \tag{2.2}$$

The problem simplifies in the limit in which $b \gg a$, that is, that we are interested in distance scales much larger than the characteristic length of the dipole. In this case we can expand the potential in Taylor series (assume $x > a$)

$$V_{\rm full}(x) = Q\left(\frac{1}{x} - \frac{1}{x-a}\right) = \frac{Q}{x}\left(1 - \frac{1}{1-a/x}\right) = \frac{Q}{x}\sum_{n=1}^{\infty}\left(\frac{a}{x}\right)^n \tag{2.3}$$

The effective description of this problem is defined by keeping only a few terms in the Taylor expansion, assuming that the rest can be neglected. For example, keeping only the first two terms, the effective dipole potential reads

$$V_{\rm eft}(x) = \frac{Q}{x}\left(\frac{a}{x} + \frac{a^2}{x^2} + \mathcal{O}\left(\left(\frac{a}{x}\right)^3\right)\right) \tag{2.4}$$

so the calculation at $x = b$ in the effective description gives keeping only the leading order

$$V_{\rm eft}(x = b) = \frac{aQ}{b^2}\,. \tag{2.5}$$

Note that we are always dealing with the same physics, but only with different descriptions. Provided we are far from the dipole, the effective potential is rather simpler while capturing at the same time all the relevant physics. Note also that $V_{\rm eff}(x)$ is only a valid description of the physics for $x \gg a$, else it gives wrong results. For example, if we try to use the effective potential for $x = a/2$ one has that $V_{\rm full}(a) = 0$ but $V_{\rm eft}(a) = 4Q/a$ instead. This shows how the effective description misses the aspects of the "full theory" which are relevant beyond the EFT regime of validity.

From this simple example we can already identify several generic features of the EFT paradigm:

- The more terms we include in the EFT expansion, the closer we will be to the full theory result.
- The EFT description is systematically improvable, and we can get as close as possible to the full result as we want (or we are able to compute).
- In general, the EFT appears conceptually simpler than the full theory, and it is often easier to compute quantities of physical interest (for example, since it contains less degrees or freedom or dynamical variables).

Similar considerations apply to many other systems, characterised by at least one pair of widely separated scales.

2.3 The quantum anharmonic oscillator

Let us now consider a second example. Assume a one dimensional quantum system consistent on a particle moving under the effects of a potential $V(x)$. The dynamics of the system are determined by the solutions of the corresponding Schroedinger equation:

$$\widehat{H}\psi(x) = E\psi(x) \qquad \widehat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\,. \tag{2.6}$$

For a generic potential this problem does not admit an exact solution. However close to a local minimum x_0 the potential can be expanded as

$$V(x) \simeq V(x_0) + \left.\frac{dV}{dx}\right|_{x=x_0}(x-x_0) + \frac{1}{2}\left.\frac{d^2V}{dx^2}\right|_{x=x_0}(x-x_0)^2 + \frac{1}{6}\left.\frac{d^3V}{dx^3}\right|_{x=x_0}(x-x_0)^3 + \ldots \tag{2.7}$$

Now the linear term vanishes at the local minimum, and if we assume that the Taylor expansion can be truncated at the second order, that is, if x is such that the following condition holds

$$\frac{1}{2}\left.\frac{d^2V}{dx^2}\right|_{x=x_0}(x-x_0)^2 \gg \frac{1}{6}\left.\frac{d^3V}{dx^3}\right|_{x=x_0}(x-x_0)^3\,, \tag{2.8}$$

then the Hamiltonian reduces to

$$\widehat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}\kappa(x-x_0)^2\,, \tag{2.9}$$

which is of course nothing but the quantum harmonic oscillator, for which the analytical solutions are known:

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right)\,, \quad \omega = \sqrt{\kappa/m}\,. \tag{2.10}$$

What happens however if we are not close enough to x_0 that the quadratic approximation holds? Do we need to compute the full solution? This is not necessary, what we need to do in the EFT spirit is to extend the validity of the model by adding additional terms in the expansion. In this case we should keep the cubic term of the potential

$$V(x) \simeq V(x_0) + \frac{1}{2}\kappa(x-x_0)^2 + \widetilde{\kappa}\,(x-x_0)^3\;, \tag{2.11}$$

which is the known as the quantum anharmonic oscillator, which has as solutions

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right)\left[1 - x_e\left(n+\frac{1}{2}\right)\right] \tag{2.12}$$

where $x_e \propto \widetilde{\kappa}$ accounts for higher order effects in the EFT expansion.

Clearly, if we could include these kind of effects to all orders, the solution of the effective theory would simply correspond to the same as in the full theory. But note the advantage of our approach: we start from simple model, that can be solved analytically, and then we systematically improve it with higher order corrections. This is the same idea that underlies the EFT paradigm, that as mentioned above are systematically improvable by adding additional orders and getting closer to the full theory.

2.4 The Fermi theory of the weak interactions

Following this two examples, we are now ready to introduce the first Effective Field Theory that we will study in these lectures: the Fermi theory of the weak interactions. Fermi theory is perhaps the paradigmatic example of an effective theory. When originally formulated, it was a phenomenological model to describe radioactive processes such as the beta decay of the neutron,

$$n \to p + e^- + \bar{\nu}_e\,. \tag{2.13}$$

At the Lagrangian level, this process takes place via a four-fermion interaction of the following form

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\left(\bar{\psi}_e\gamma^\mu(1+\gamma_5)\psi_\nu\right)\left(\bar{\psi}_n\gamma^\mu(1+\gamma_5)\psi_p\right)\;. \tag{2.14}$$

Note that proton and neutrons appear here as elementary fermions, since their quark substructure was unknown by the time the theory was formulated.

We can learn a lot about the consequences of this theory by simple dimensional analysis, a tool was also famously exploited by Fermi. In this course we will mostly work in natural units, $\hbar = c = G = 1$, so the action S has mass-dimensions of zero, $[S]=0$, and therefore since $S=\int d^4x\mathcal{L}$ we see that the mass-dimensions of the Lagrangian must be four, $[\mathcal{L}]=4$. Now, if we take into account that the mass dimension of the fermion field is $[\psi]=3/2$ (which you get from the Dirac equation, which contains a term $\mathcal{L}_{\rm dirac} \in m\bar{\psi}\psi$), when we see that the coupling constant of the theory G_F, known as Fermi constant, is not dimensionless but rather dimensionful, $[G_F]=-2$, whose value fixed from the experiment is $G_F = 1.166\times 10^{-5}$ GeV^{-2}.

This theory described pretty well all available data from neutron beta decays. However, it was unknown what mechanism fixed the value of the Fermi constant G_F, or in other words, the mechanism that fixes the rate of the weak radioactive processes? Perhaps G_F was just one of a random parameter fixed by anthropic considerations?

In addition to this conceptual conundrum, there was also another concern about the validity if Fermi theory, related to the behaviour of scattering amplitudes at high energies. Basic crossing considerations in QFT indicate that if the decay $n \to p + e^- + \bar{\nu}_e$ was allowed by the theory, then the scattering process $\nu_e + n \to e^- + p$ should also be present, as depicted in Fig. 2.2. Based on dimensional analysis, one can see that the cross-section for $\nu_e + n \to e^- + p$ scattering should scale as $\sigma \propto G_F^2$, given that the amplitude is proportional to $\mathcal{A} \propto G_F$. Recalling that a cross-section is nothing but an area, its mass-dimensions are $[\sigma] = -2$ and thus purely based on dimensional reasons we can derive that at large energies the scattering cross-section scales as

$$\sigma\left(\nu_e + n \to e^- + p\right) \propto G_F^2 E_{\rm cm}^2 \,, \tag{2.15}$$

where $E_{\rm cm}$ is the center of mass energy, and we assume that $E_{\rm cm}$ is large enough that all masses can be neglected (and thus $E_{\rm cm}$ is the only magnitude in the problem with the right units to obtain dimensionally consistent expressions).

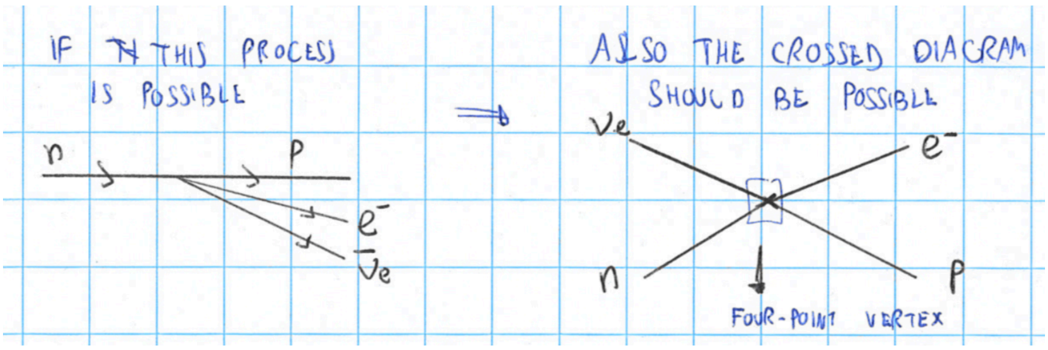


Figure 2.2: The existence of the neutron beta-decay implies, from general QFT principles, that the $\nu_e + n \to e^- + p$ scattering process should also exist.

Eq. (2.15) indicates that the scattering cross-section grows quadratically with the energy. This is potentially a serious problem: if we go to high enough energies, the theory will violate unitarity, meaning that the probability for this scattering process will become larger than one. So this scaling with the energy indicates that there is something weird in Fermi theory: it seems unlikely that it is a fundamental theory valid at all energies. On the other hand, it describes well the physics around the proton and neutron mass scales, so surely we don't want to throw it away entirely.

Can we estimate the regime of validity of Fermi theory? Assume that we can compute one loop corrections in the theory. From dimensional considerations, we have that the cross-section will look like

$$\sigma\left(\nu_e + n \to e^- + p\right) = c_1 G_F^2 E_{\rm cm}^2 + c_2 G_F^3 E_{\rm cm}^4 \,, \tag{2.16}$$

where c_1, c_2 are dimensionless numbers expected to be of $\mathcal{O}(1)$, and the $\mathcal{O}(G_F^2)$ term arises from the interference between the Born and the one-loop amplitude. For this expansion to have physical sense, the one-loop term should be smaller than the Born contribution, and this means

$$\frac{c_2 G_F^3 E_{\rm cm}^4}{c_1 G_F^2 E_{\rm cm}^2} \lesssim 1 \quad \to \quad G_F E_{\rm cm}^2 \lesssim 1 \tag{2.17}$$

which implies that $E_{\rm cm} \lesssim \sqrt{1/G_F} \simeq 300$ GeV, and thus Fermi theory fails to provide reliable predictions above that scale.

And thus our concise inspection of Fermi theory has revealed two main questions to be addressed:

- What mechanism determines the value of G_F?
- How we can make sense of the high energy behaviour of the theory above $E_{\rm cm} \lesssim \sqrt{1/G_F} \simeq 300$ GeV?

Of course we know the answers to these two questions: Fermi theory is not a fundamental QFT but rather an effective theory, whose regime of validity is limited by some energy cutoff Λ. And we also know that the corresponding fundamental theory is nothing but the Standard Model, and that $\Lambda \simeq M_W$ is the mass of the W boson that in Fermi theory has been integrated out, as represented schematically in Fig. 2.3

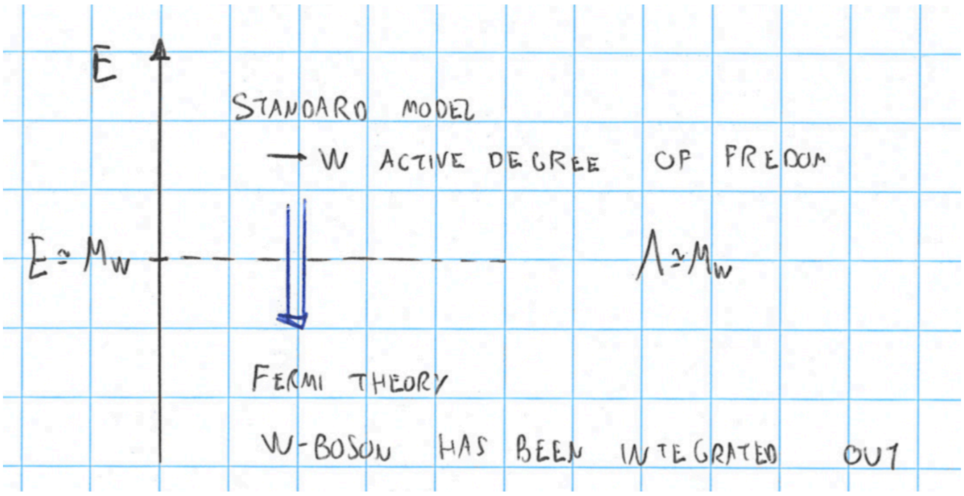


Figure 2.3: Fermi theory is an EFT with the SM as full theory, after the electroweak gauge bosons W, Z are integrated out at low energies.

In the full theory, the Standard Model, the scattering process $\nu_e + n \to e^- + p$ proceeds via the exchange of a W^+ gauge boson. However at low energies the W boson propagator collapses to a single point, and therefore one ends up with the four-fermion interaction characteristic of Fermi theory.

So now we have the answer to one of the questions that were raised above: the theory is well-behaved at large energies since there one has only renormalizable interactions where the W boson couples to two fermions. To answer the other question about Fermi theory, namely what mechanism determines the value of G_F, we need to introduce a very important concept of the EFT framework.

Matching in EFTs

By *matching* we denote the mechanism whereby some or all the parameters of the effective theory are computed in terms of those of the full theory. The matching procedure can be carried out in different ways, for example by computing scattering amplitudes for the same process in the full and in the effective theories and then imposing that the two amplitudes are equivalent.

Let us illustrate how the concept of EFT matching works in the case of Fermi theory. As we have mentioned above, the scattering amplitude in Fermi theory at the Born level will be proportional to

$$\mathcal{A}_{\rm eft}(\nu_e + n \to e^- + p) \propto G_F \, , \tag{2.18}$$

while in the full theory, the Standard Model, as indicated by the diagram in Fig. 2.4, the scattering amplitude for the same process will be proportional to the square of the electroweak $f\bar{f}W$ coupling, $g_{\rm EW}$, and to the W boson propagator

$$\mathcal{A}_{\rm full}(\nu_e + n \to e^- + p) \propto \frac{g_{\rm EW}^2}{k_W^2 - M_W^2} \, . \tag{2.19}$$

In general $\mathcal{A}_{\rm eft}$ and $\mathcal{A}_{\rm full}$ will be different, since the two theories are not valid in the same kinematic range (in particular the EFT is only valid at low energies). But in the region of validity of Fermi theory, defined by the condition, $E \ll M_W$, then the two amplitudes should indeed be identical since they correspond to the same process. This implies in the regime of validity of the EFT the amplitude in the full theory reduces to

$$\mathcal{A}_{\rm full}(\nu_e + n \to e^- + p) \propto \frac{g_{\rm EW}^2}{M_W^2} + \mathcal{O}\left(M_W^{-4}\right) \, , \tag{2.20}$$

since all momentum transfers are well below the cutoff, $k^2 \ll M_W^2$.

By imposing the equivalence between $\mathcal{A}_{\rm eft}$ and $\mathcal{A}_{\rm full}$ in the appropriate kinematical regime we thus find the *matching conditions* of the theory

$$G_F \propto \frac{g_{EW}^2}{M_W^2} \, , \tag{2.21}$$

whereby the parameters of the EFT (l.h.s.) are expressed in terms of the couplings and masses of integrated-out particles of the full theory (r.h.s.). Note also that for $g_{\rm EW} = \mathcal{O}(1)$ we see that $M_W \simeq G_F^{-1/2}$, in good agreement with our previous estimate of the EFT cutoff scale Λ where the EFT description stops being valid. In other words, theoretical inspection of Fermi theory revealed that "new physics" should appear at the 100 GeV scale, and indeed this was confirmed with the discovery in 1983 of the W and Z bosons.

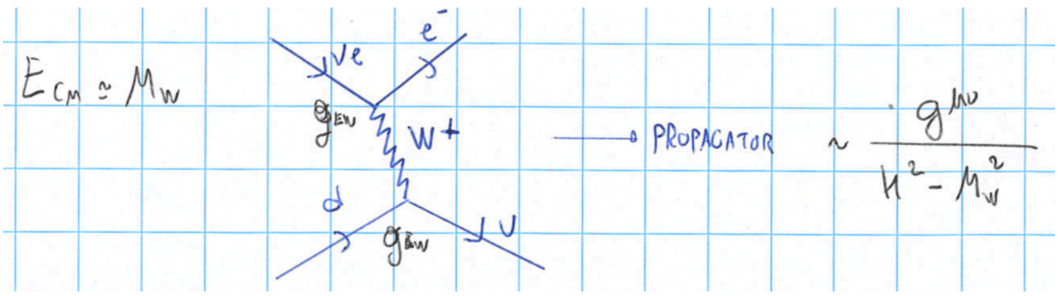


Figure 2.4: The SM diagram that contributes to the matching with Fermi theory via the four-point scattering amplitude.

The matching relation in Eq. (2.21) illustrates a crucial feature of EFTs: the information about the integrated-out particles (in this case the W boson: its coupling to fermions $g_{\rm EW}$ and its mass) are still present in the low-energy EFT though in an indirect way, as encoded by its parameters. Note that in many cases, as was the case originally with Fermi theory, one does not have access to the full theory, and thus the structure of the EFT Lagrangian must be constructed from general considerations (field content and symmetries) with parameters fixed from the experiment. As will be discussed later, this is exactly the situation that one encounters when the Standard Model is treated as an EFT.

2.5 Bottom-up vs Top-down in scalar EFTs

Let us provide another example of a relatively simple EFT. In this case, one considers a scalar QFT that contains two massive scalar fields, denoted by ϕ and Φ, and that is defined by the Lagrangian:

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int}\,, \tag{2.22}$$

where the kinetic term is given by

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2\,. \tag{2.23}$$

What can we say about the interaction term? If we had no information about the contents of $\mathcal{L}_{\rm int}$, we could use the so-called bottom-up approach to construct EFTs:

Bottom-up QFT construction

Within a bottom-up approach, the Lagrangian of a QFT is constructed including all possible operators with the appropriate field content that satisfy all the symmetries of the theory (for example Lorentz of gauge invariance). In such a case, the operators will be proportional to unknown coefficients to need to be extracted from experimental data. Note that these considerations apply both to "full" QFTs as well as to EFTs.

If we assume that $\mathcal{L}_{\rm full}$ corresponds to a renormalizable theory, it can only contain operators with mass-dimension less or equal than four. This requirement that the theory is renormalizable leaves us with only two types of interaction operators: those can contain three fields (and thus that are not invariant under parity) and those that contain four fields, and thus are invariant under a parity transformation. With these considerations, we can write the most general interaction term of this scalar QFT as follows

$$\mathcal{L}_{\rm int} = \mathcal{L}_1 + \mathcal{L}_2\,, \tag{2.24}$$

where the parity-conserving interaction Lagrangian is given by

$$\mathcal{L}_1 = c_1\phi^4 + c_2\phi^2\Phi^2 + c_3\Phi^4 + c_4\phi^3\Phi + c_5\phi\Phi^3\,, \tag{2.25}$$

whose coupling constants are dimensionless, while the operators that contribute to the parity-violating interaction Lagrangian are given by

$$\mathcal{L}_2 = d_1\phi^2\Phi + d_2\Phi^2\phi + d_3\phi^3 + d_4\Phi^3\,, \tag{2.26}$$

whose couplings have mass-dimension of $[d_i] = +1$. If we impose that our theory must be invariant under parity transformations ($\phi \to -\phi$ and $\Phi \to -\Phi$) then the $d_i = 0$ coefficients must vanish.

Now we shall assume the specific case where the interaction term is composed by the following two operators:

$$\mathcal{L}_{\rm int} = -\frac{1}{4}\kappa\phi^2\Phi^2 - \frac{1}{3!}\rho\phi^3\Phi\,. \tag{2.27}$$

In the regime where $E \ll M$, we can integrate out the heavy field Φ and construct an effective field theory whose only dynamical degree of freedom is the light scalar field ϕ. This theory can be written as

$$\mathcal{L}_{\rm eft} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \mathcal{L}_{\rm eft,int}\,. \tag{2.28}$$

We need to determine now the interaction Lagrangian of the EFT and to match its coefficients to those of the full theory. First of all we note that the UV-complete Lagrangian conserves parity, so this implies that the EFT Lagrangian should also be invariant under this symmetry. This is an important consideration that holds when constructing EFTs:

Symmetries in the EFT

In general, the full and effective theories will share the same symmetries: this is required to ensure that in the appropriate energy regime scattering amplitudes computed with both theories coincide. The exception is those symmetries related to heavy degrees of freedom integrated out, where the EFT will only exhibit the residual symmetry.

Imposing parity conservation, the most general form of the interaction term for our effective theory where the heavy field Φ has been integrated out will be

$$\mathcal{L}_{\rm eft,int} = c_4\phi^4 + c_6\phi^6 + c_{2,2}\phi^2\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) + c_8\phi^8 + c_{4,2}\phi^4\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) + c_{0,4}\left(\partial_\mu\phi\right)^4 + \ldots$$

where the dots indicate higher-dimension operators, which will be not relevant for the following discussion.

Now we need to perform the matching between the masses and couplings in the full theory with the EFT coefficients. You can convince yourselves that at three level there are no diagrams in the full theory that can generate a non-zero c_4 or a non-zero c_8 coefficient. We can instead generate a non-zero value of c_6 by performing the matching using the six-point function, see Fig. 2.5, which in the full theory is given by

$$\mathcal{M}_{\rm full}\left(\phi\phi\phi\to\phi\phi\phi\right) \propto \frac{\rho^2}{M^2+q^2} \simeq \frac{\rho^2}{M^2}\left(1-\frac{q^2}{M^2}+\ldots\right)\,, \tag{2.29}$$

where we have expanded the propagator in a series in $q^2/M^2 \ll 1$, while the same amplitude in the EFT has the leading expression

$$\mathcal{M}_{\rm eft}\left(\phi\phi\phi\to\phi\phi\phi\right) \propto c_6\,, \tag{2.30}$$

and therefore from the matching conditions we can determine that

$$c_6 \propto \frac{\rho^2}{M^2}\,. \tag{2.31}$$

Furthermore, from the full theory we can see that the next term in the $1/M^2$ expansion in the six-point amplitude is proportional to two powers of the momentum flowing through the heavy field. Momentum-dependent terms in the EFT arise via derivative couplings, and thus one can note that the operator proportional to $c_{4,2}$ (which has two derivative couplings but also contains six fields) also contributes to $\mathcal{M}_{\rm eft}$ and from the matching conditions it is possible to determine that

$$c_{4,2} \propto \frac{\rho^2}{M^4}\,. \tag{2.32}$$

Note that $[c_{4,2}]=-4$, as expected from the power counting in the EFT.

All other terms in the EFT interaction term listed above vanish due at three-level. However, at the one-loop level we can generate new terms in the EFT interaction term. For example, you can convince yourselves that the $c_4\phi^4$ term can be generated at the one-loop level from diagrams in the full theory such as those shown below. In this case we can state that $c_4 \propto \kappa^2$, but we cannot say much more about the scaling with M given that in one-loop matching we cannot use simple dimensional analysis (since loop integrals are involved). Further, the same diagram can potentially contribute to the $c_{2,2}$ operator, which also contributes to the four-point function in the EFT and includes derivative couplings. In addition, the six-point function used for the three level matching before will also receive contributions at the one-loop level via the diagram displayed below. This diagram upon matching will lead to $\mathcal{O}\left(\kappa^3\right)$ corrections to both the c_6 and $c_{4,2}$ coefficients.

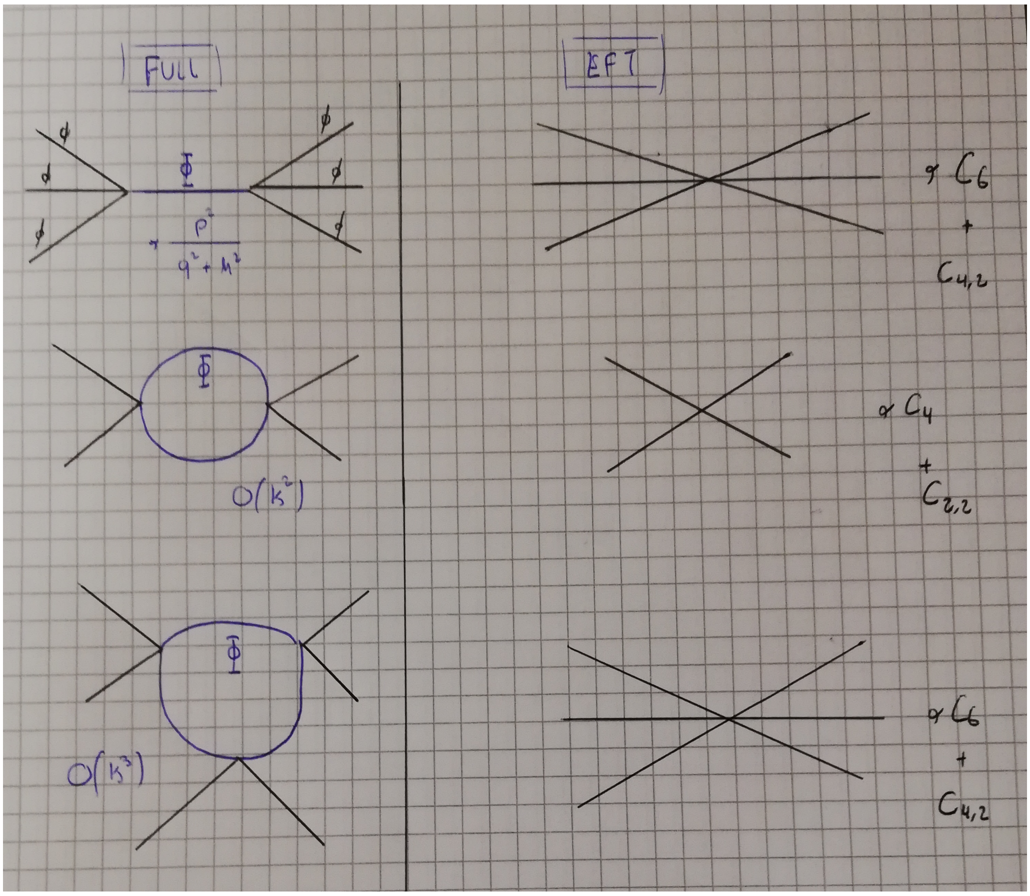


Figure 2.5: The diagrams in the full and effective theories that contribute to their matching at the tree and loop levels. See text for more details.

In these lectures we will focus mostly on tree level matching, with matching at the one-loop level being discuss at the purely qualitative level.

Exercise I.1

Consider a scalar QFT that contains two massive scalar fields ϕ and Φ and is defined by the Lagrangian:

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int}\,, \tag{2.33}$$

where the kinetic term is given by

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2\,. \tag{2.34}$$

and the interaction term by

$$\mathcal{L}_{\rm int} = -\frac{1}{2}b\phi^2\Phi - \frac{1}{4!}c\phi^4\Phi\,. \tag{2.35}$$

i) By integrating out the heavy field Φ, construct the most general EFT for the light field ϕ in the limit $E \ll M$, making sure that the parity symmetry of ϕ is maintained. **ii)** By means of tree-level matching, estimate the values of the three non-zero EFT couplings with the lowest mass-dimension. **iii)** Can one consider the original theory as UV-complete? Why? **iv)** Are here loop effects relevant for the matching between the full and effective theories?

2.6 An EFT with Yukawa interactions

To wrap up this lecture, we can consider another example of a relatively simple EFT. This time the starting point is a UV-complete theory containing fermion and scalar fields coupled via a Yukawa-type interaction. This theory is defined by the following Lagrangian

$$\mathcal{L}_{\rm full} = i\bar\psi\gamma^\mu\partial_\mu\psi + \frac{1}{2}\left(\partial_\mu\Phi\right)^2 - \frac{M^2}{2}\Phi^2 - \lambda\bar\psi\psi\Phi\,. \tag{2.36}$$

As we can read from the Lagrangian, this theory contains a massless fermion field ψ and a massive scalar field Φ, coupled together by means of a Yukawa interaction with dimensionless coupling λ.

We will try now to construct the corresponding EFT which arises by integrating out the heavy scalar field Φ. In other words, we want a theory that describes the physics at $E \ll M$ without making explicit reference to Φ. The only dynamical degree of freedom in this theory will thus be the massless fermion field ψ. You can convince yourselves that the EFT that one obtains after integrating out the heavy field Φ is given by

$$\mathcal{L}_{\rm eft} = \bar\psi\gamma^\mu\partial_\mu\psi + c_4\left(\bar\psi\psi\right)^2\,, \tag{2.37}$$

that is, a massless fermion with quartic self-interactions (similar to the coupling that defined Fermi theory). In order to determine the value of the EFT coupling c_4 in terms of the parameters of the full theory, we need to carry out the matching procedure. In the full theory, the four-point amplitude will be proportional to

$$\mathcal{A}_{\rm full}(\psi\psi\to\psi\psi) \propto \lambda^2\frac{1}{k^2-M^2} \simeq \frac{\lambda^2}{M^2}\left(1+\frac{k^2}{M^2}+\mathcal{O}\left(\frac{k^4}{M^4}\right)\right)\,, \tag{2.38}$$

where k^μ is the four-momentum flowing through the heavy scalar line in the full theory. By computing the corresponding amplitude in the EFT we determine that $c_4 \propto \lambda^2/M^2$, see also Fig. 2.6. Note that the EFT amplitude receives contributions from both the s-channel and the t-channel scattering in the full theory, which in general have associated different kinematical dependences.

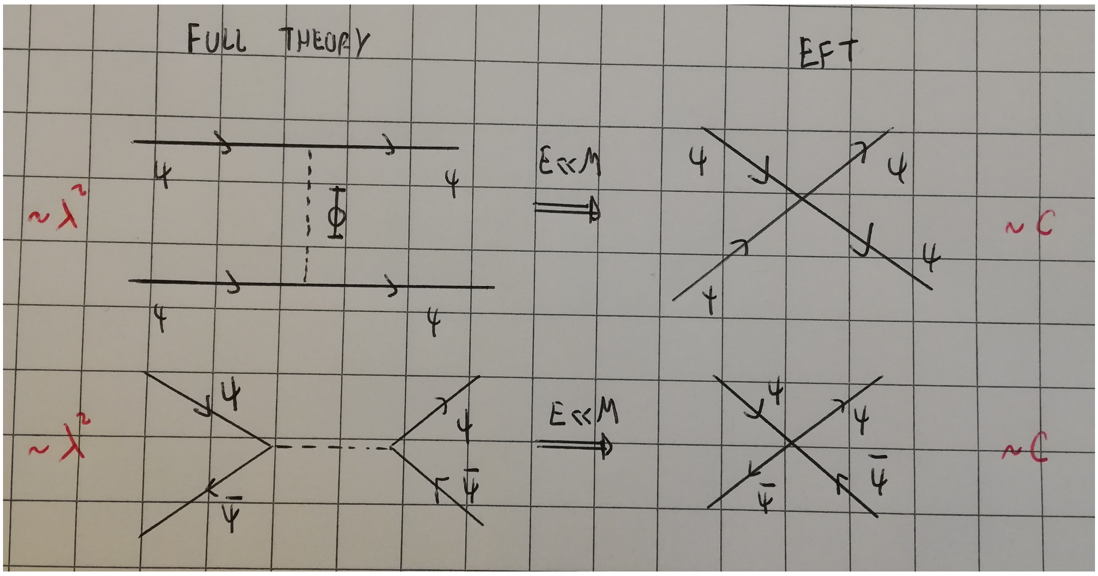


Figure 2.6: Tree-level EFT matching in the Yukawa-like scalar/fermion theory.

This way we achieve the EFT matching at the lowest order in the EFT expansion. How we can account higher orders in the $1/M$ expansion? What needs to be done is to include in the matching the first subleading term in Eq. (2.38), proportional to $\lambda^2 k^2/M^4$. To reproduce this term in the EFT, we need to add a new dimension-8 operator in the interaction Lagrangian, which includes derivative couplings to the massless fermion field to generate a momentum-dependent interaction vertex. It can be shown that we can reproduce this higher order term in the full theory by extending the EFT as follows

$$\mathcal{L}_{\rm eft} = \bar{\psi}\gamma^\mu\partial_\mu\psi + c_4\left(\bar{\psi}\psi\right)^2 + c_{2,2}\left(\partial^\mu\bar{\psi}\partial_\mu\psi\right)\left(\bar{\psi}\psi\right) \,, \tag{2.39}$$

where the matching conditions require $c_{2,2} \propto \lambda^2/M^4$, with the two derivative couplings ensuring a term proportional to k^2 in the scattering amplitude.

To summarise, after tree-level matching has been performed, the EFT Lagrangian reads

$$\mathcal{L}_{\rm eft} = \bar{\psi}\gamma^\mu\partial_\mu\psi + \widetilde{c}_4\frac{\lambda^2}{M^2}\left(\bar{\psi}\psi\right)^2 + \widetilde{c}_{2,2}\frac{\lambda^2}{M^4}\left(\partial^\mu\bar{\psi}\partial_\mu\psi\right)\left(\bar{\psi}\psi\right) \,, \tag{2.40}$$

where now $\widetilde{c}_4$ and $\widetilde{c}_{2,2}$ are computable, $\mathcal{O}(1)$ coefficients and from where we can visualize the EFT as a series expansion in inverse powers of the mass of the heavy scalar field Φ.

2.7 Running and mixing in EFTs

Let us consider a general EFT constructed from n massless scalar fields $\{\phi_i\}$ and a heavy scalar field Φ with mass M upon integrating out the latter. Following our discussion above, we know that the most general form of the interaction Lagrangian of this EFT will take the form

$$\mathcal{L}_{\rm int,eft} = \sum_{i=1}^{N_{d4}} c_i^{(4)}\mathcal{O}_i^{(4)} + \sum_{j=1}^{N_{d5}} \frac{c_j^{(5)}}{M}\mathcal{O}_j^{(5)} + \sum_{k=1}^{N_{d6}} \frac{c_k^{(6)}}{M^2}\mathcal{O}_k^{(6)} + \ldots \tag{2.41}$$

where $c_i^{(d)}$ are the so-called *Wilson coefficients* of the EFT, $\mathcal{O}_j^{(d)}$ are the N_d operators with mass-dimension d constructed from the light fields of the theory, and where we have introduced suitable powers of M to make all coefficients dimensionless. The values of the

Wilson coefficients $\{c_i^{(d)}\}$ are to be determined from the matching of scattering amplitudes in the full theory, or to be measured from the experiment if the latter is unknown. The dots in Eq. (2.41) correspond to higher order terms in the EFT expansion, suppressed at least by factors of M^{-3}.

Up to know in our discussions we have assumed that the EFT coefficients such as $\{c_i^{(d)}\}$ are constants. However we know that in general due to the renormalisation of UV divergences coupling constants in QFT acquire a dependence with the energy, think for example of the coupling constant of the strong interaction $\alpha_S(E)$. Therefore in general the Wilson coefficients c_i in the EFT Lagrangian will not be constant but rather dependent on the energy of the process $c_i(E)$: in QFT jargon, we say that the coefficients now *run with the energy*. Schematically, at the first non-trivial level an EFT coupling constant c in a renormalizable QFT will acquire a dependence of the energy of the process E of the following form

$$c(E) = \frac{c_0}{1 - \gamma_c \frac{c_0}{16\pi^2} \ln \frac{E^2}{E_0^2}} \,, \tag{2.42}$$

where γ_c is called the *anomalous dimension of the coupling* and can be computed order by order in perturbation theory. In this expression $c_0 \equiv c(E_0)$ is the value of the coupling at some reference energy E_0 which is typically determined from the experiment. An interesting feature of Eq. (2.42) is that there is a value of the energy for which the coupling constant becomes divergent, and that is given by the following condition:

$$1 = \gamma_c \frac{c_0}{16\pi^2} \ln \frac{E^2}{E_0^2} \quad \to \quad E = E_0 \exp\left(\frac{8\pi^2}{\gamma_c c_0}\right) \,. \tag{2.43}$$

The physical interpretation of this result is the following:

The Landau pole

In QFTs with running couplings, we denote as the *Landau pole* as the value of the energy of the process for which the running coupling diverges. The presence of the Landau pole indicates the breakdown of the perturbative description of the QFT, and demands the use of non-perturbative methods in the relevant kinematic region.

Depending on the sign of the anomalous dimensions γ_c associated to this specific operator, we will encounter two different qualitative behaviors. First of all for $\gamma_c < 0$ we have an *asymptotically free theory* with a coupling that vanishes in the ultraviolet and blows up in the infrared as the Landau pole is approached. In this case the infrared behaviour of the QFT is dominated by non-perturbative dynamics. On the other hand, for $\gamma_c > 0$ the theory becomes trivial in the infrared (where the coupling vanishes) but becomes strongly coupled in the ultraviolet once the energy is close to the Landau pole. These two behaviors are depicted in Fig. 2.7.

While these properties are generic features of QFTs, it is important to recall them here since in our study of the SMEFT we will often deal with measurements separated by a wide range of energies. In such case a consistent interpretation of the experimental data requires to properly account for the *running of the Wilson coefficients* with the energy, that determine the relative strength of each of the operators that define the EFT.

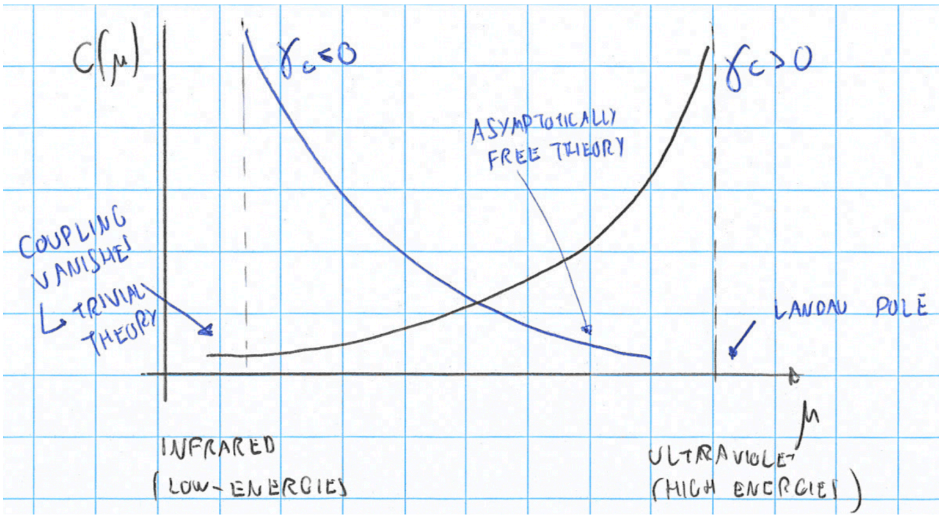


Figure 2.7: Schematic running of the couplings in QFTs.

Exercise II.1

Consider a Wilson coefficient $c(E)$ in an EFT whose dependence with the energy is determined by the following equation

$$\frac{d\,c(E)}{d\ln E} = \gamma_c\,(c(E))^n\;, \tag{2.44}$$

Solve this equation for $n = 0, 1, 2$ and 3 considering both the cases where γ_c is positive and negative. Sketch the qualitative behaviour of the running of the Wilson coefficient in each case, and determine where there is a Landau pole.

In addition to inducing the scale-dependence of the Wilson coefficients, the QFT renormalisation process also induces the so-called *mixing* between different operators. This means that in general the effect of a variation of the energy in the process under consideration is not limited to a rescaling of the couplings (the Wilson coefficients), for example in the case of dimension-6 operators

$$\sum_{j=1}^{N_{d6}} \frac{c_j^{(6)}(\mu)}{\Lambda^2}\mathcal{O}_j^{(6)}(\mu) \neq \sum_{j=1}^{N_{d6}} \frac{c_j^{(6)}(\mu')}{\Lambda^2}\mathcal{O}_j^{(6)}(\mu')\,, \tag{2.45}$$

but rather one finds a different linear combination of operators as the energy is varied

$$\sum_{j=1}^{N_{d6}} \frac{c_j^{(6)}(\mu)}{\Lambda^2}\mathcal{O}_j^{(6)}(\mu) = \sum_{j=1}^{N_{d6}} \frac{b_j^{(6)}(\mu')}{\Lambda^2}\mathcal{O}_j^{(6)}(\mu')\,, \tag{2.46}$$

where now the coefficients b_j are a linear combination of the original c_j coefficients. This has the important consequence that even if some coefficients $c_j(\mu) = 0$ vanish at a given energy, it is possible that $b_j(\mu') \neq 0$ at some other energy due to operator mixing effects. In other words, some operators might not contribute to a given process at one energy but will instead contribute to the same process if the energy is varied.

The renormalisation group

In quantum field theories, the dependence on the scale of both operators and Wilson coefficients (running) as well as their mixing is determined by Renormalisation Group Equations (RGE). The structure of these RGEs is determined by the renormalisation conditions of the theory, and depend on anomalous dimensions (also known as *beta functions*) that can be computed in perturbation theory.

In general for a QFT (either a full or an effective theory) that is characterised by n different scale-dependent operators $c_i(\mu)$, the associated RGE equations will be given by

$$\frac{d}{d\ln\mu}c_j(\mu) = \gamma_{ij}c_i(\mu)\,, \tag{2.47}$$

with γ_{ij} being the matrix of anomalous dimensions that determines the mixing of operators between different scales μ. These type of equations are extremely important in the context of the SMEFT, specially when we aim to connect predictions for processes that exhibit a large separation in energies, such as Higgs produciton on the one hand and the decay of B-mesons (quark-antiquark bound states containing a bottom quark) on the other hand.

Let us consider an explicit example, in the same scalar theory that we used above. The full theory was defined by

$$\mathcal{L}_{\rm full} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 + \mathcal{L}_{\rm int}\,, \tag{2.48}$$

which for a specific choice of the interaction terms had associated the following effective Lagrangian when the heavy field Φ was integrated out:

$$\mathcal{L}_{\rm eft} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4!}c_4\phi^4 + \frac{1}{6!}\frac{c_6}{M^2}\phi^6\,, \tag{2.49}$$

where the dimensionless Wilson coefficients c_4 and c_6 are determined by the couplings and masses of the heavy field Φ that has been integrated out at low energies. Now consider the 4-point scattering amplitude $\mathcal{A}(\phi\phi \to \phi\phi)$. At tree level there is a single diagram that contributes to the amplitude in the EFT, proportional to c_4, as shown in Fig. 2.8. However at the one loop level we have different diagrams that contribute to the four-point amplitude, some of them that involve also c_4, but others that involve instead the other coupling of the theory c_6. The two loop diagrams above are divergent, implying that the renormalisation of c_4 will involve *both* the contribution from c_4 and c_6, and leading to renormalisation group equations that mix the contributions from the two operators. Therefore the explicit calculations of those diagrams would allow the evaluation of the anomalous dimension matrix in Eq. (2.47)

One can carry out the calculation of the one-loop matrix of anomalous dimensions explicitly and then one finds that

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} c_4(\mu) \\ c_6(\mu)\end{pmatrix} = \begin{pmatrix} \gamma_{44} & \gamma_{46} \\ \gamma_{64} & \gamma_{66}\end{pmatrix}\begin{pmatrix} c_4(\mu) \\ c_6(\mu)\end{pmatrix}. \tag{2.50}$$

For simplicity, assume that the γ_{ij} matrix is constant and that at some reference scale μ_0 the coefficient associated to the quartic coupling vanishes, $c_4(\mu_0) = 0$. By diagonalizing the matrix of anomalous dimensions one finds that the general solution for c_4 is given by

$$c_4(\mu) = c_4(\mu_0)e^{\widetilde{\gamma}_4\ln\mu/\mu_0} + \left(c_6(\mu) - c_6(\mu_0)\right)e^{\widetilde{\gamma}_6\ln\mu/\mu_0} \tag{2.51}$$

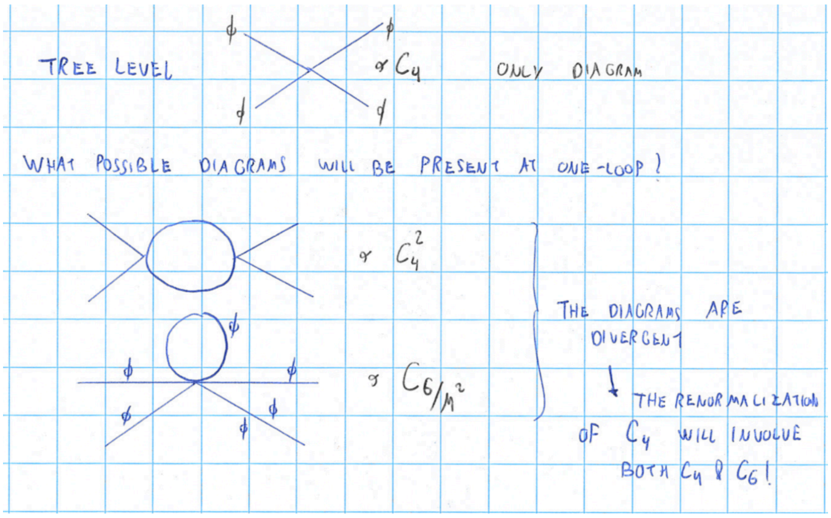


Figure 2.8: The origin of mixing at the one loop level in a scalar EFT.

which can also be written as

$$c_4(\mu) = c_4(\mu_0)\left(\frac{\mu}{\mu_0}\right)^{\widetilde{\gamma}_4} + \left(c_6(\mu) - c_6(\mu_0)\right)\left(\frac{\mu}{\mu_0}\right)^{\widetilde{\gamma}_4}, \tag{2.52}$$

which shows that even if $c_4(\mu_0) = 0$ vanishes at some reference scale, in general due to operator mixing effects one will have that $c_4(\mu) \neq 0$. This shows that in the context of an EFT an statement such as "the value of this Wilson coefficient is X" is not meaningful unless the corresponding value for the energy is specified.

2.8 Redundancy in EFTs

The discussions so far highlight that EFTs share a number of characteristic properties:

- **Decoupling.** The heavy fields present in the full theory decouple in the low energy limit. The reason is that in this limit $M \to \infty$ and the heavy field does not play any role in the EFT: it cannot be produced on-shell and is very suppressed off-shell. The only remnant of its existence is through the values of the EFT Wilson coefficients determined via the matching procedure.

- **Locality.** The Lagrangian of an EFT can only be composed by *local* operators. For example, in an EFT with a single light scalar field $\phi(x)$ the only operators allowed are those that involve positive powers of the field and of its derivatives, such as $\partial_\mu\phi(x)$, $\partial_\nu\partial_\mu\phi(x)$ and so on. Non-local operators cannot be used in the EFT construction.

- **Predictivity.** Effective field theories are predictive at any given order in the expansion in $1/M$, provided that the number of observables used to fix the Wilson coefficients is equal or larger than the number of operators. In this respect EFTs are equally predictive than full QFTs when restricted to the appropriate range of validity.

In addition to these defining properties, another generic feature of EFTs is the existence of some degree of **redundancy** in its formulation. This means that in general different combinations of operators in the Lagrangian of the theory will end up leading to exactly the same physical predictions. Let us consider to illustrate this point the following Lagrangian:

$$\mathcal{L}_{\text{eft}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda_4}{4!}\phi^4 + \Delta\mathcal{L}^{(i)}\,, \tag{2.53}$$

where $\phi(x)$ denotes as usual a real scalar field. We see that in addition to the quartic coupling proportional to λ_4 there is an additional interaction term denoted by $\Delta\mathcal{L}^{(i)}$. It is possible to demonstrate that three possible choices for $\Delta\mathcal{L}^{(i)}$ lead exactly to the same amplitudes. These three options are:

- $\Delta\mathcal{L}^{(1)} = -3\phi^2(\partial_\mu\phi\partial^\mu\phi)$.
- $\Delta\mathcal{L}^{(2)} = \phi^3\nabla\phi$, with the D'Alambertian operator being $\nabla = \partial^\mu\partial_\mu$
- $\Delta\mathcal{L}^{(3)} = -m^2\phi^4 - \lambda\phi^6$

This shows that the formulation of $\mathcal{L}_{\text{eft}}$ is not unique but that there exhibits a certain a degree of redundancy: the same physics can be formulated by different combinations of higher-dimensional operators. This is a generic feature of EFTs which is also present in the case of the SMEFT, as we will see shortly. For example, we can show how $\Delta\mathcal{L}^{(1)}$ and $\Delta\mathcal{L}^{(2)}$ are equivalent to each other. By using partial integration, one can write

$$\int d^4x\phi^3\nabla\phi = \int d^4x\phi^3\partial^\mu\partial_\mu\phi = \phi^3\partial\phi\Big|_\infty - \int d^4x 3\phi^2\left(\partial_\mu\phi\partial^\mu\phi\right)\,, \tag{2.54}$$

where the surface term at infinity vanishes due to the boundary conditions (the fields vanish at infinity). Therefore from the point of view of the action of the theory, which determines the equations of motion, using either $\Delta\mathcal{L}^{(1)}$ or $\Delta\mathcal{L}^{(2)}$ leads to exactly the same results. One can also show that $\Delta\mathcal{L}^{(2)}$ and $\Delta\mathcal{L}^{(3)}$ are equivalent by means of using field redefinitions and the equations of motion.

3 The Standard Model Effective Field Theory

After this general discussion about the properties and features of effective field theory, we are ready to present the main topic of these lectures, namely the SMEFT. First of all we will remind the reader of the general structure of the Standard Model, in particular of its field content, symmetries, and interactions. Next we will extend the SM with higher-dimensional operators, built upon the same fields and symmetries of the SM, and discuss a bit what are the effects of these operators. We will then consider two specific examples of relevant SMEFT operators. First of all, we will match a UV-complete theory with a new heavy Z' boson to the SMEFT, and study the implications of these effects for high-energy lepton-proton scattering. Second, we will study how SMEFT operators affect a precision SM process, the decay of the top quark. A more detailed discussions of the phenomenological implications of the SMEFT will then be presented in Sect. 4.

3.1 The Standard Model in a nutshell

Let us start with a brief reminder of the main ingredients that enter the construction of the Standard Model of particle physics. This exercise will also be useful to set the notation and conventions that will be adopted in the rest of the lectures. The Lagrangian of the Standard Model can be written as

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm EW} + \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm H} \,, \tag{3.1}$$

as a sum of the QCD, electroweak, and Higgs contributions, although of course this separation is somewhat arbitrary. For example, the QCD Lagrangian is given by

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_i^{(f)} \left(i\gamma_\mu D^\mu_{ij} - m_f \delta_{ij} \right) \Psi_j^{(f)} - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} \tag{3.2}$$

where the sum runs over all quarks and the field strength tensor contains among others the self-interactions between the gluons. The QED Lagrangian is part of the EW Lagrangian. In a while we will review the basic structure of the electroweak and the Higgs components of the Lagrangian, which provide important information about how we should construct the SMEFT operators.

What are the principles that define the Standard Model? Its building blocks are the following:

- *The field content*: first of all we need to specify the kind of fields that are present in our theory: in the SM case this includes six quarks, three charged leptons, and three massless neutrinos. All matter particles are represented by fermion fields. In principle one needs to separate right-handed from left-handed fermions, which have associated different properties.

- *The symmetries that these fields must satisfy*: in particular the SM fields satisfy a $\mathrm{SU}(3)\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ gauge symmetry, where each field is endowed with a specific set of gauge charges. After electroweak symmetry breaking, this gauge symmetry is reduced to $\mathrm{SU}(3)\otimes\mathrm{U}(1)_Q$ Other symmetries of the SM include Lorentz symmetry and CPT symmetry.

  Note that in addition to these *fundamental* symmetries that SM has also associated *accidental* symmetries, such as baryon and lepton number conservation. Accidental symmetries do not arise from fundamental conservation principles but rather from the specific field content of the theory.

- *How these symmetries are realised and/or broken*: for instance, in the SM the $SU(2)_L \otimes U(1)_Y$ symmetry is spontaneously broken due to the structure of the Higgs sector, in particular of the Higgs potential. Note that the interactions of the Higgs boson with the SM particles are not a direct consequence of the gauge principle but rather of the chosen mechanism for electroweak symmetry breaking.

- *The renormalizability properties*: finally, we need to specify whether or not the theory is UV-complete, that is, that can be extrapolated up to arbitrarily large energies being able to provide meaningful physical predictions.

Operator classification. The latter condition in the list above, renormalisation, is at the root of the conceptual differences between the understanding the SM either as a fundamental or as an effective theory. To illustrate this point, it us useful to recall that in QFT we can classify operators according to their mass dimension. The interactions between quantum fields can be classified into three different types, depending on the *mass dimensions* of the relevant coupling constant:

- *Relevant interactions*, when the coupling constant has positive mass dimensions, that is, where $[g] > 0$.

  An example of this is the mass term in scalar QFT, $-m^2\phi^2/2$, where the "coupling" has mass dimensions of $[g] = 2$. Another examples are the vacuum energy term (cosmological constant) or part of the Higgs Lagrangian.

- *Marginal interactions*, when the coupling constant is dimensionless $[g] = 0$.

  This is the case for example of the quartic interaction in $\lambda\phi^4$ theory in $d = 4$ dimensions. This is also the case for all interaction terms in the SM.

- *Irrelevant interactions*, when the coupling constant has negative mass dimensions $[g] < 0$.

  An example of this is Fermi's theory of beta decay, whose Lagrangian includes a four-fermion interaction term of the form

  $$\mathcal{L}_F \supset G_F \bar{\psi}\psi\bar{\psi}\psi \,, \tag{3.3}$$

  and thus a massive coupling constant with negative mass dimensions, $[G_F] = -2$.

You can check that all terms in the Lagrangian have mass-dimension 4: for example the covariant coupling between fermions and gauge fields is $\left[\bar{\psi}A\psi\right] = 2 \otimes [\psi] + [A] = 2 \otimes 3/2 + 1 = 4$. In particular the fact that $\mathcal{L}_{\rm SM}$ is composed only by operators with mass-dimension of at most 4 implies that the SM is a renormalizable QFT, valid up to arbitrarily high scales, at least up to $E \simeq M_{\rm plack}$ where quantum gravity effects should set in. Indeed, in general we can have the following classification of QFTs:

- A *renormalizable theory* includes only *relevant* or *marginal* operators. This implies that a finite set of counterterms if enough to remove all the UV divergences.

- A *non-renormalizable theory* contains at least *one irrelevant operator*. This implies that to remove all infinities it would be necessary to include an infinite number counterterms.

A renormalizable theory can in principle be used to make predictions up to arbitrarily high scales, hence it is known as a UV-complete theory. However, this is a theoretical prejudice: while EFTs contain irrelevant operators, there are perfectly *bona fide* QFTs provided that we make sure we only apply them in their regime of validity. In this respect, if we assume that the SM is a QFT valid not until very high scales $E \simeq M_{\rm plack}$ but rather only up to $E \simeq \Lambda$ where Λ is not too far from the electroweak scale, it makes sense to treat it not as a UV-complete theory but rather as an EFT.

The EW & Higgs Lagrangian in the SM. In addition to the QCD Lagrangian, Eq. (3.2), the SM also contains the electroweak and Higgs sector, where the latter is responsible for spontaneously breaking the electroweak symmetry exhibited by the former. As in the case of QED and QCD, electroweak interactions can be described by a renormalizable QFT with a Lagrangian which is invariant under a specific type of gauge transformations. In particular, electroweak interactions are invariant under the $SU_L(2)\otimes U(1)$ gauge group. In $SU_L(2)\otimes U(1)$ group, the first subgroup corresponds to the *weak isospin* quantum number and the second subgroup to the *weak hypercharge*.

To construct the electroweak Lagrangian, one can start by assuming that we have a purely bosonic theory, and write down a Lagrangian which exhibits this specific gauge invariance, in analogy with QED and QCD. Our Lagrangian will now have three massless bosons W_i, associated to the weak isospin subgroup, and one more boson, B, charged under the weak hypercharge group. As for the photon and gluon, these bosons are massless due to gauge invariance constraints. The resulting Lagrangian will be

$$\mathcal{L} = -\frac{1}{4}W^{i,\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\,, \tag{3.4}$$

with $i = 1, 2, 3$, and where the corresponding field strength tensors have the familiar expressions

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_W\epsilon^{ijk}W^j_\mu W^k_\nu\,, \tag{3.5}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \tag{3.6}$$

with g_W the coupling associated to the weak isospin interaction, and where we have used that the structure constants of $SU_L(2)$ are the totally antisymmetric tensor ϵ^{ijk}. The fact that $SU(2)$ is non Abelian leads to the self-interactions of the W^i_μ bosons, as was the case for the gluons in QCD. The Lagrangian Eq. (3.4) cannot describe the weak interaction, which is short-ranged because of the large mass of the force carriers. One needs to introduce a mass term without spoiling gauge symmetry. The way to achieve this is by means of *spontaneous symmetry breaking*.

The coupling of the gauge fields to matter, as usual, will be through the covariant derivative, which for the case of the $SU_L(2)\otimes U(1)$ gauge group reads

$$D^\mu = \delta_{ij}\partial^\mu + ig_W\left(T\cdot W^\mu\right)_{ij} + iY\delta_{ij}g'_W B^\mu\,, \tag{3.7}$$

where g'_W is the weak hypercharge coupling and Y is the value of the so-called *weak hypercharge* of a given matter particle. The matrices T are a suitable representation of the $SU(2)$ algebra. The $SU(2)$ algebra is defined by

$$\left[T^i, T^j\right] = i\epsilon^{ijk}T^k\,, \tag{3.8}$$

and typically one defines the combination

$$T^\pm = T^1 \pm iT^2\,. \tag{3.9}$$

Note that in the full SM the covariant derivative Eq. (3.7) acting on the fermion fields also contains the interactions between the quarks and gluons via the strong interaction as dictated by SU(3) color gauge symmetry.

In the Standard Model, the electroweak $SU(2) \otimes U(1)$ gauge symmetry group is spontaneously broken by the *Higgs mechanism*. This requires to add a new complex doublet of scalar fields,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{3.10}$$

which transform as a doublet of $SU_L(2)$ and have weak hypercharge value of $Y = 1/2$. It is possible to couple the Higgs field ϕ to the massless electroweak Lagrangian while preserving gauge symmetry by means of the same covariant derivative that was introduced in Eq. (3.7), namely

$$\begin{aligned} \mathcal{L} &= D_\mu \phi^\dagger D^\mu \phi - -\mathcal{V}\left(\phi^\dagger\phi\right) = \\ & \left(\partial^\mu \phi^\dagger + i g_W \left(T \cdot W^\mu\right)\phi^\dagger + i\frac{1}{2}\delta_{ij} g'_W B^\mu \phi^\dagger\right) \cdot \\ & \left(\partial_\mu \phi - i g_W \left(T \cdot W_\mu\right)\phi - i\frac{1}{2}\delta_{ij} g'_W B^\mu \phi\right) - \mathcal{V}\left(\phi^\dagger\phi\right) \end{aligned} \tag{3.11}$$

where for simplicity the isospin indices have been left implicit. It is easy to check explicitly that the above Lagrangian is invariant under $SU_L(2) \otimes U(1)$ transformations.

A remarkable feature of Eq. (3.11) is the presence of a potential term for the scalar field. Other than the fact that it can depend only on the product $\phi^\dagger\phi$, to enforce gauge symmetry, there is no other restriction on its form.

In the Standard Model, the Higgs potential takes the following value

$$\mathcal{V}\left(\phi^\dagger\phi\right) = \lambda\left(\phi^\dagger\phi\right)^2 - \mu^2\left(\phi^\dagger\phi\right). \tag{3.12}$$

The Higgs potential Eq. 3.12 is the only *ad-hoc* component of the Standard Model, whose shape is not determined by any symmetry principle. Other than minimality, there is no principle that selects Eq. (3.12) as compared to other potentials that can also break EW symmetry.

There are some interesting feature of this potential:

- the sign of the mass term is different as the one that would be used for example in the $\lambda\phi^4$ scalar theory. This causes the field to have a *vacuum expectation value* $\langle\phi\rangle \neq 0$ at the minimum.
- the λ accounts for the self-interactions of the scalar field ϕ with itself.

Therefore $\lambda, \mu^2 > 0$, this potential exhibits degenerate minima for values of the field ϕ which are different from zero, as shown in Fig. 3.1: this is the famous *Mexican hat* potential of the Higgs field. Remarkably, as we will show, single Higgs measurements only probe the region of the Higgs potential close to the minimum. To reconstruct the full potential, measuring double Higgs production is necessary, this situation is represented in Fig. 3.2.

In order to better represent the phenomenon of electroweak symmetry breaking, it useful to make explicit the four real components of the Higgs doublet as follows:

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{3.13}$$

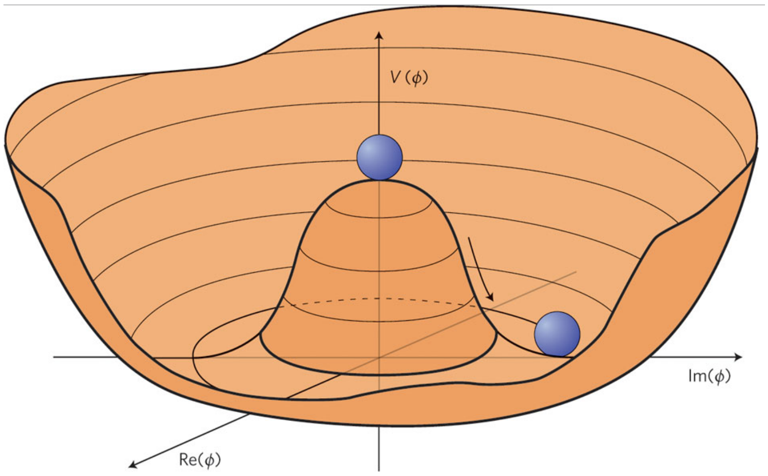


Figure 3.1: The Higgs field potential for a choice of parameters so that $\lambda, \mu^2 > 0$.

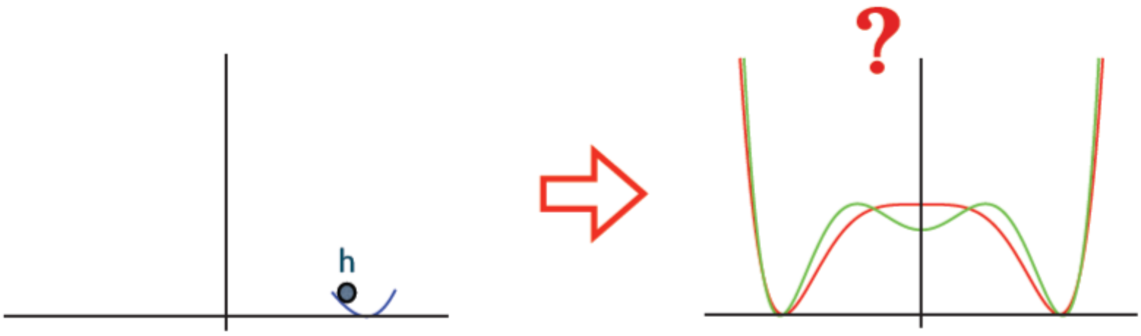


Figure 3.2: Single Higgs measurements only probe the region of the Higgs potential close to the minimum (the vacuum expectation value). To reconstruct the full potential, measuring double Higgs production is necessary [**?**].

so that the inner product of Higgs fields reads

$$\phi^\dagger \phi = \sum_{i=1}^{4} \phi_i^2 \,, \tag{3.14}$$

which is of course invariant under four dimensional rotations. By minimizing the classical potential, we find that the space of minima is degenerate, and defined by the condition

$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \,, \tag{3.15}$$

and transformation of the Higgs field that satisfy this condition therefore have no energy costs associated.

Since all minima that satisfy the above equation are equivalent, it is possible to select as a particular choice

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{3.16}$$

This is called a vacuum expectation value, or vev. While this choice is arbitrary, we should ensure that it is invariant under residual $U(1)$ hypercharge transformations. We realize that with this choice, transformations generated by the specific combination T^3+Y leave the vacuum expectation value invariant, that is

$$\left(T^3+Y\right)\langle\phi\rangle = 0 \tag{3.17}$$

This combination is the single unbroken generator that is associated with the *electric charge*

$$Q \equiv T^3+Y = \left(\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right) , \tag{3.18}$$

which arises because

$$T^3 = \left(\begin{array}{cc} 1/2 & 0 \\ 0 & 1/2 \end{array}\right) , \quad Y = \frac{1}{2}\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) , \tag{3.19}$$

since the Higgs boson has $Y=1/2$ weak hypercharge.

The specific choice of vacuum expectation value (vev) that we have adopted breaks the $SU_L(2)\otimes U(1)$ symmetry, since we have identified a preferred direction in the internal space. This can be easily seen: under a generic $SU(2)$ transformation, the vev Eq. (3.16) would be transformed into a different vacuum expectation value. The Higgs mechanism is the paradigmatic example of *spontaneous symmetry breaking*: the underlying theory respects the symmetry but the ground state does not. As we now show, this allows to generate a mass for the massive weak bosons without breaking gauge symmetry.

At this point we need to reparametrize the scalar field in order to be able to represent the fluctuations with respect to the vacuum expectation value of the Higgs field. We can introduce the following parametrization

$$\phi = U^{-1}(\xi)\left(\begin{array}{c} 0 \\ (h+v)/\sqrt{2} \end{array}\right) \tag{3.20}$$

where we have introduced the following unitary matrix

$$U(\xi) \equiv \exp\left(-i\frac{T\cdot\xi}{v}\right) . \tag{3.21}$$

Now we have four degrees of freedom, three in ξ and one in h, corresponding to the four original degrees of freedom of the complex scalar doublet. Is clear that if now we set to zero these field fluctuations we reproduce the original vev value. Note also that Eq. (3.21) has the same form as a $SU(2)$ gauge transformation. But since our theory is gauge invariant, we are free to perform arbitrary gauge transformations without modifying the physics of our theory. So therefore let's us perform the following $SU(2)$ gauge transformation of the Higgs field

$$\begin{array}{rcl} \phi & \to & U(\xi)\phi \\ T\cdot W^\mu & \to & UT\cdot W^\mu U^{-1} + \frac{i}{g_W}\left(\partial^\mu U\right)U^{-1} , \end{array} \tag{3.22}$$

the second equation has of course a closely related expression as the gluon gauge transformation under the color group $SU(3)$. Upon this gauge transformation, the ξ_i degrees of freedom no longer appear in the Higgs Lagrangian, since we cancel them from ϕ and the rest of the electroweak Lagrangian is invariant under gauge transformations. They will re-appear latter as the longitudinal models of the massive gauge bosons. This gauge is known as the *unitary gauge*.

Following the Higgs field redefinition, and in the unitary gauge, the Higgs Lagrangian now reads

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h - \mathcal{V}\left((v+h)^2/2\right) \\ &+ \frac{(v+h)^2}{8}\chi^\dagger \left(2g_W\left(T\cdot W_\mu\right) + g'_W B_\mu\right)\left(2g_W\left(T\cdot W^\mu\right) + g'_W B^\mu\right)\chi\,, \end{aligned} \tag{3.23}$$

where we have defined $\chi \equiv (0,1)$, a unit vector in the direction of the vev. In addition to Eq. (3.23), we also have in the theory the kinematic massless gauge boson term, Eq. (3.4), which are left unaffected by either adding the Higgs filed to the theory or by the gauge transformation.

Let us take a closer look at the consequences of spontaneous symmetry breaking. In the Higgs Lagrangian, we can consider only the terms that are quadratic in the vector boson fields. Using the expressions for the generators of $SU(2)$,

$$T_1 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}\,, \qquad T_2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix}\,, \qquad T_3 = \begin{pmatrix} 1/2 & 1 \\ 0 & -1/2 \end{pmatrix}\,, \tag{3.24}$$

we obtain the following result

$$\mathcal{L} \supset \frac{v^2}{8}\left[\left(g_W W^3_\mu - g'_W B_\mu\right)\left(g_W W^{\mu,3} - g'_W B^\mu\right)\right] + 2g_W^2 W^-_\mu W^{\mu,+}\,, \tag{3.25}$$

where we have defined the following combination of the W gauge boson fields:

$$W^\pm_\mu \equiv \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}}\,. \tag{3.26}$$

Now, we want to express the quadratic terms of the Lagrangian so they they are diagonal in the fields, since physical fields propagate independently. Note also that the two fields B and W^3 are electrically neutral, since they vanish under the action of the electric charge operator Q. Therefore, it is possible to rotate the electrically neutral fields into new fields which are diagonal in the Lagrangian, and thus that propagate independently, using the following condition:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}\,, \tag{3.27}$$

where the degree of mixing of the two fields is determined by the ratio of the coupling constants of the $SU(2)$ weak isospin and $U(1)$ weak hypercharge:

$$\sin^2\theta_W \equiv \frac{g'^2_W}{g'^2_W + g^2_W}\,. \tag{3.28}$$

It is now easy to see how the quadratic terms of the Lagrangian transform under the rotation Eq. (3.27) and we get that the term quadratic in vector boson fields reads now

$$\mathcal{L} = \frac{v^2 g_W^2}{4} W^-_\mu W^{\mu,+} + \frac{\left(g'^2_W + g^2_W\right)^2 v^2}{8} Z_\mu Z^\mu\,, \tag{3.29}$$

and there is no mass term for the photon A_μ as expected since we want to recover QED. Therefore, the Higgs mechanism allows to give masses to the physical W and Z fields

$$M_W = \frac{1}{2} v g_W\,, \quad M_Z = \frac{1}{2} v\sqrt{g_W^2 + (g'_W)^2}\,. \tag{3.30}$$

The result is the net effect of electroweak symmetry breaking, which we summarize now:

- We start with a QFT of gauge fields invariant under $SU(2) \otimes U(1)$. Gauge fields are massless due to gauge invariance.
- We have added a complex scalar doublet ϕ, which couples to the W, B fields though the covariant derivative as requested by gauge invariance.
- This field ϕ has a potential which exhibits degenerate minima for $\phi \neq 0$. Choosing a particular direction in the space of minima, and expanding the field around it, we obtain terms which are quadratic in the W, B fields.
- Diagonalizing the propagators, we find that of the two rotated fields, one, the photon A_μ is massless and the other, the Z boson, acquires a mass $M_Z = v\sqrt{g_W^2 + g_W'^2}/2$.

The Higgs sector. Following the electroweak symmetry breaking process, of the initial four components of the complex doublet scalar ϕ only one turns out to be physical, the Higgs boson h, while the other three are substituted by the longitudinal modes of the now massive gauge bosons. If we take the Lagrangian, consider only the Higgs sector, and expand the potential, we find the following terms

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h - \mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 \,, \tag{3.31}$$

and therefore the mass of the Higgs boson is $M_h = \sqrt{2}\mu = \sqrt{2\lambda}v$, as can be read from the kinetic term. Since the mass of the W and Z bosons is known, v has been known for a long time, and the only other free parameter related to the Higgs sector is μ, its mass. Once the Higgs mass is measured, there are no other free parameters in the electroweak Lagrangian.[1]. Note also that at the LHC we have only probed the first two terms in Eq. (3.31). In order to measure the coefficient of the third term, and thus to really reconstruct the shape of the Higgs potential, we need to measure double Higgs production.

A natural consequence of the Higgs mechanism is that it provides the mass for the fermionic fields, and what this implies for the Yukawa coupling between the Higgs boson and all massive fermions. The key requirement is that the complete Standard Model Lagrangian must be invariant under $SU_L(2) \otimes U(1)$. As we will show now, this implies that any explicit mass term in the Lagrangian is forbidden, and thus one needs to generate a mass through spontaneous symmetry breaking as well.

In order to satisfy gauge invariance, bosonic fields must couple to the matter (fermion) fields using the covariant derivative. Before electroweak symmetry breaking, the electroweak Lagrangian with fermions contains terms of the form

$$\begin{aligned} \mathcal{L} \supset\, & \bar{\psi}_R \left(\gamma^\mu \partial_\mu + i g_W' Y_R \gamma^\mu B_\mu\right) \psi_R + \\ & \bar{\psi}_R \left(\gamma^\mu \partial_\mu + i g_W \gamma^\mu T \cdot W_\mu + i g_W' Y_L \gamma^\mu B_\mu\right) \psi_R \,, \end{aligned} \tag{3.32}$$

where we have used the fact that $SU(2)_L$ rotations only affect left handed fermions, and that weak hypercharge is different for left handed and right handed fermions. The values of the weak hypercharge of right and left-handed fermions are fixed by their electric charge, $Q = T^3 + Y$, and for the SM matter particles their values are summarized in Table 3.1. Before electroweak symmetry breaking, all the SM fermions are massless, since a explicit mass term would violate $SU(2)_L \otimes U(1)$ symmetry. This allows to treat separately left-handed and right-handed fermions in terms of their weak charges. As we see in Table 3.1, left-handed fermions correspond to a $SU_L(2)$ doublet, while right-handed quarks are singlets, and there are no right-handed neutrinos (at least in the SM).

[1] We are not considering the interactions with fermionic matter for the time being.

Fermion	T^3_L	Y_L	T^3_R	Y_R	Q_f
up quark	1/2	1/6	0	2/3	2/3
down quark	1/2	1/6	0	2/3	2/3
electron neutrino	1/2	-1/2	-	-	0
electron	-1/2	-1/2	-	-1	-1

Table 3.1: The weak isospin and weak hypercharge for the first family of right- and left-handed fermions in the Standard Model. The same assignments hold for the second and third families.

Let us now show how the Higgs mechanism, which gave mass to W and Z bosons, can also give mass to the fermions. Let us now add to the weak Lagrangian the so-called *Yukawa interaction* terms, of the form

$$\mathcal{L} \supset \sum_f g_f \bar{\psi}_f \psi_f \phi \,, \tag{3.33}$$

where the sum runs over all the massive fermions in the SM. This term is gauge invariant, and after electroweak symmetry breaking, when we have

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix} \tag{3.34}$$

we find that fermions acquire a mass $m_f = g_f v/\sqrt{2}$. In addition, the Yukawa interaction term Eq. (3.33) also dictates the strength of the interaction between the Higgs boson and the fermions, since now the Lagrangian contains

$$\mathcal{L} \supset \sum_f \left(\frac{\sqrt{2} m_f}{v} \right) \bar{\psi}_f \psi_f h \,, \tag{3.35}$$

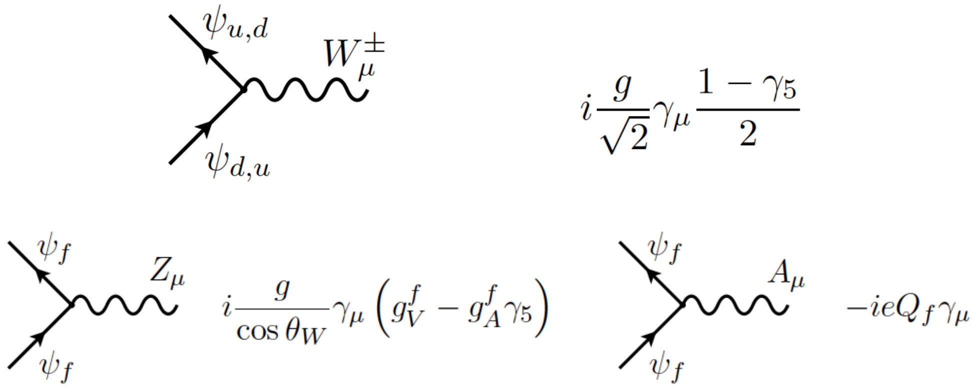
so therefore the interaction between the Higgs boson and the SM fermions is proportional to the fermion mass. It is worth noting that in the SM the values of the Yukawa couplings g_f are not fixed by any principle, but are free parameters of the theory that need to be determined experimentally. However we have the specific prediction that fermionic masses are proportional to the interaction strength of the same fermions with the Higgs boson.

Concerning the interactions between the fermions and the weak bosons, we can start from the Lagrangian Eq. (3.32) and perform the transformation into the physical weak bosons, $W^\pm$ and Z^0, as well as the photon, as done before. When this is performed, we find that the weak Lagrangian contains the following interaction terms:

$$\begin{aligned} \mathcal{L} \supset & \sum_f \bar{\psi}_f \left(i\gamma^\mu - m_f - g_W \frac{m_f h}{2 m_W} \right) \psi_f \\ - & \frac{g_W}{2\sqrt{2}} \sum_f \bar{\psi}_f \left(\gamma^\mu (1-\gamma_5) T^+ W^+_\mu + \gamma^\mu (1-\gamma^5) T^- W^-_\mu \right) \psi_f \\ - & e \sum_f q_f \bar{\psi}_f \gamma^\mu A_\mu \psi_f - \frac{g_W}{2\cos\theta_W} \sum_f \bar{\psi}_f \gamma^\mu \left(V_f - A_f \gamma_5 \right) \psi_f Z_\mu \,. \end{aligned} \tag{3.36}$$

From this Lagrangian, we can read the values of the coupling constants between the SM fermions and the W, Z bosons. In the case of the Z boson, the specific values of the coupling depend on the fermion, while for W the coupling is the same for all (left-handed) fermions. For the Z boson, we have that the vector (V_i) and axial-vector (A_i) couplings are given by

$$V_f = T^3_f - 2 Q_f \sin^2 \theta^2_W \,, \quad A_f = T^3_f \,. \tag{3.37}$$



$$i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2}$$

$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right) \qquad -ieQ_f\gamma_\mu$$

$$g_V^f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W\,, \quad g_A^f = \frac{1}{2}T_f^3\;.$$

Figure 3.3: Feynman diagrams for the interactions between fermions and weak gauge bosons.

The purely electromagnetic interaction is also reproduced. While at very high energies the weak and electromagnetic coupling are of the same order, at low energies the effective weak coupling appears to be much smaller, since it is suppressed by the large masses of the weak gauge bosons. In Fig. 3.3 we show the Feynman rules that allow to compute scattering amplitudes involving the interaction of fermions and weak gauge bosons. This information will be useful later in the lectures, where we will work out the effects of a new heavy gauge boson Z' at the level of the SMEFT.

3.2 Basic features of the SMEFT

Following this brief reminder of the main features of the Standard Model, we can now move to construct the SMEFT. Recall the previous discussion about the defining principles of the SM: we have to impose or establish first of all:

- *The field content.*
- *The symmetries that these fields must satisfy.*
- *How these symmetries are realised and/or broken.*
- *The renormalizability properties*: the theory should be valid to provide reliable predictions up to arbitrarily high energies.

We will now exclude the last condition: imposing that the SM must to be valid up to arbitrarily high scales is a theoretical prejudice. We know that it provides a valid description of physical processes for energies up to several TeV, but we cannot say much more about higher energies at least from a model-independent perspective.

Therefore, instead of requiring that the SM is valid up to the Planck scale, let's demand instead that it is valid only up to some cutoff scale $\Lambda \gg m_h$, which must be well above the electroweak scale since else the new physics effects would have already been visible. All other principles of the SM, in particular field content and the associated symmetries, must be kept unchanged. In this case there is no need to restrict ourselves to marginal (dimension 4) operators, and one should extend our Lagrangian with *irrelevant* higher-dimensional operators. We can thus write down the SMEFT Lagrangian as follows

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_{i=1}^{N_{d5}} c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_{i=1}^{N_{d6}} c_i^{(6)}\mathcal{O}_i^{(6)} + + \frac{1}{\Lambda^3}\sum_{i=1}^{N_{d7}} c_i^{(7)}\mathcal{O}_i^{(7)} + \dots \quad (3.38)$$

where we have that:

- Λ is the EFT cut-off scale: only process which involve momentum transfers $E \ll \Lambda$ can be reliable described by the EFT. This scale Λ can be interpreted as the mass scale where new particles and interactions, beyond those included in the EFT, become important and need to be explicitly accounted for.

  Furthermore, Λ also represents the EFT expansion parameter, which allows organizing different terms by order of relevance: the higher the power of Λ, the more suppressed the contribution of the associated operator will be.

- The $\{c_i^{(d)}\}$ are the Wilson coefficients of the EFT expansion, or in other words, the coupling constants of the EFT theory. They are fixed by the couplings and masses of the full theory, in the same was as G_F was fixed by $g_{\rm EW}$ and M_W in the full theory. In the case where the corresponding full theory is unknown, as happens with the SMEFT, then the coefficients $\{c_i^{(d)}\}$ are free parameters of the EFT that needs to be constrained by experimental data.

- The $\mathcal{O}_i^{(d)}$ indicate operators of mass-dimension d, which are constructed with the fields that already appear in the SM. The SMEFT contains N_{d5} operators of mass-dimension five, N_{d6} operators of mass-dimension six, and so on. The number of operators is determined by the field content and the symmetries of the EFT [3] and grows very rapidity as d increases.

- The dots in Eq. (3.38) indicate higher order terms in the $1/\Lambda$ expansion, the first neglected one being dimension-8 operators suppressed as $1/\Lambda^8$.

Note that there are very specific rules about which of the higher-dimensional operators $\mathcal{O}_i^{(d)}$ are allowed and which ones are not:

> Not every possible operator that we can construct with the SM fields is allowed in the SMEFT: only those combinations of fields that satisfy the same symmetries as the SM (such as gauge and Lorentz invariance) can be meaningfully added to the SMEFT Lagrangian.
> In addition, one can impose additional symmetry requirements at the SMEFT level which are not part of the SM fundamental symmetries, such as lepton and baryon number conservation or cancellation of tree-level flavour-changing neutral currents, which are properties of the SM but arising from accidental symmetries and cancellations due to its specific field content.

What are thus the main conceptual and practical advantages of formulating the SM as an Effective Field Theory, Eq. (3.38)? One can consider several benefits, including:

- The SMEFT provides a model-independent parametrisation of physics beyond the SM. Any UV-complete theory of BSM physics that reduces to the SM at low energies can be described by it. This implies that bounding the Wilson coefficients of the SMEFT $\{c_i^{(d)}\}$ with experimental data implies bounding at once a huge set of possible UV completions of the SM.

- The SMEFT correlates BSM corrections affecting in different sectors of the SM, such as the Higgs and top quark sector with the bottom quark sectors.

  An important example, which we will discuss later in these lectures, are the flavour non-universality anomalies in the decays of bottom mesons reported by LHCb: using the SMEFT one can predict how models that explain these anomalies affect Higgs and other high-p_T observables.

- The SMEFT is systematically improvable in perturbation theory, and in particular corrections in the strong and weak couplings α_S and α_W can be evaluated. Furthermore, one can enhance the regime of validity of the EFT approximation by including higher-order operators in the $1/\Lambda$ expansion.

The goal of these lectures is to present an overview of what are the implications of this SMEFT formalism for the phenomenology of the Large Hadron Collider and other experiments.

EFT corrections to SM processes. We would now like to discuss what are the practical consequences of extending the SM with higher dimensional operators as we did in Eq. (3.38). In particular, we would like to assess how these new SMEFT operators modify scattering amplitudes and decay rates of SM process. For this purpose, we will restrict ourselves in the following to dimension-six operators, which are typically the most relevant ones in the context of collider applications.

In this case, the SMEFT Lagrangian will be given by

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{j=1}^{N^{(d6)}} \frac{c_j^{(6)}}{\Lambda^2} \mathcal{O}_j^{(6)} \,, \tag{3.39}$$

and we will consider for simplicity a single operator, so that dropping the indices we obtain

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{c_1}{\Lambda^2} \mathcal{O}_1 \,. \tag{3.40}$$

Now we need to compute the scattering amplitude for a SM process that is also sensitive to the operator $\mathcal{O}_1$, in particular by having the appropriate field content and quantum numbers. We can write this SMEFT-enhanced amplitude as

$$\mathcal{A}_{\rm tot} = \mathcal{A}_{\rm SM} + \mathcal{A}_1 \,, \tag{3.41}$$

where the SMEFT contribution $\mathcal{A}_1 \propto c_1$ is proportional to the associated Wilson coefficient c_1.

Naively one could expect that the associated effects at the cross-section level would be

$$\sigma_{\rm tot} = \sigma_{\rm SM} + \frac{c_1^2}{\Lambda^4} \sigma_{11} \,, \tag{3.42}$$

where σ_{11} arises from squaring $\mathcal{A}_1$. But however in general this is not the case, since one needs to account the effects of the possible *interference* between the SM and the SMEFT amplitudes. These interference effects introduce $\mathcal{O}\left(\Lambda^{-2}\right)$ corrections to the cross-section which are often the phenomenologically dominant ones, and thus we instead we should express the SMEFT-modified cross-section as

$$\sigma_{\rm tot} = \sigma_{\rm SM} + \frac{c_1}{\Lambda^2} \sigma_1 + \frac{c_1^2}{\Lambda^4} \sigma_{11} \,. \tag{3.43}$$

It is also worth mentioning that in some cases there symmetry considerations for which the interference term vanishes, σ_1 and the dominant SMEFT contribution arises at the quadratic level.

Interference effects in the SMEFT

Basic QFT considerations indicate that the scattering amplitude for a given process, for example top pair production, $\mathcal{A}(gg \to t\bar{t})$, one should sum over all possible contributions. In the SM one has the couplings $g_s\, gt\bar{t}$ and $g_s\, ggg$, so one contribution proceeds via $gg \to g \to t\bar{t}$ and is $\propto g_s^2$, see also Fig. 3.5.
In the SMEFT, one $d = 6$ operator induces a vertex $c_{tG} ggt\bar{t}/\Lambda^2$, so another contribution proceeds via $gg \to t\bar{t}$ and is proportional to c_{tG}/Λ^2. Since in both contributions the field content and quantum numbers of the initial and final state are the same, the two amplitudes will interfere when computing the cross-section.

Let us for the time being restrict to the $\mathcal{O}\left(\Lambda^{-2}\right)$ SMEFT-induced corrections, which are often the most relevant in phenomenology (since they are less suppressed as compared to the corresponding $\mathcal{O}\left(\Lambda^{-4}\right)$ effects). Following the above considerations, one can show that the general expression for the cross-section can be written as

$$\sigma_{\rm tot}(E) = \sigma_{\rm SM}(E) \times \left[1 + \sum_{i=1}^{N_{d6}} \kappa_i c_i^{(6)} \frac{v^2}{\Lambda^2} + \sum_{i=1}^{N_{d6}} \widetilde{\kappa}_i c_i^{(6)} \frac{E^2}{\Lambda^2} + \mathcal{O}\left(\Lambda^{-4}\right) \right] , \tag{3.44}$$

where E is the characteristic energy of the scattering process. In this equation $v = 246$ GeV is the Higgs boson vacuum expectation value, and $\widetilde{\kappa}_i$ and κ_i are (dimensionless) process-dependent coefficients. One can see we have two qualitatively different contributions to the cross-section as a function of the process energy:

- Terms proportional to the Higgs vev v^2. These are energy-independent corrections, so their effect is the same no matter how energetic the collision is. These effects are suppressed as v^2/E^2.
- Terms proportional to E^2: these terms grow fast with the energy of the process. These effects are suppressed as E^2/Λ^2, and thus for experiments that are probing energy scales above the electroweak region $v \simeq E$, such as the Large Hadron Collider, the effects might dominate over the previous category.

EFT validity

Recall that the SMEFT is only valid for momentum transfers well below the scale where New Physics sets in, $E \ll \Lambda$. When carrying out a SMEFT study, one should verify that the data included always satisfies this condition, else the EFT analysis is not consistent.

Furthermore, it is worth mentioning that we have already seen these energy-growing effects: they also arise in Fermi theory, and they remind us that the EFT has a restricted range of validity, and that it cannot be extrapolated to arbitrarily high energies. These E^2/Λ^2 are a consequence of the fact that our QFT contains irrelevant operators and thus its high-energy behaviour is ill-defined.

From the structure of Eq. (3.44) we can appreciate the two main ingredients that are needed to evaluate the SMEFT corrections to a generic cross-section:

- The Wilson coefficients $\{c_i^{(6)}\}$ and the cut-off scale Λ are process-independent, and their values are fixed by matching with the full theory. In the case the full theory is unknown, as happens with the SMEFT, both $\{c_i^{(6)}\}$ and Λ are to be treated as free parameters to be extracted from experimental data.

> Note that a phenomenological analysis can only extract from the experimental data the $c_i^{(6)}/\Lambda^2$ combination, and cannot separate the Wilson coefficient from the cutoff scale Λ. Therefore one has to *interpret* the results for specific values of Λ, always ensuring that one works in the regime of validity of the EFT.

- The cross-sections $\widetilde{\kappa}_i$ and κ_i are process dependent, and can be computed by deriving the Feynman rules associated to the operators $\{\mathcal{O}_i^{(6)}\}$ and applying them to the specific progress under consideration. They depend also on the details of the experimental analysis, such as the cuts and the binning of the distributions.

After this general introduction to the main properties of the SMEFT, we discuss illustrative cases of some of the higher-dimensional operators that compose it.

3.3 The SMEFT higher-dimensional operators.

Let us now provide some examples of the kind of higher-dimensional operators that arise in the SMEFT. Recall that as discussed above these operators are build from the SM fields and they must satisfy their symmetries. These operators are written before electroweak symmetry breaking, thus satisfying explicitly their SU(3)$\otimes$SU(2)$_L\otimes$U(1)$_Y$ gauge symmetry and in including the Higgs doublet Subsequently one needs to evaluate their form after EW symmetry breaking, where they satisfy only the SU(3)$\otimes$U(1)$_Q$ residual gauge symmetry and include the Higgs field as explicit degree of freedom.

In Table we summarise the dimension-6 SMEFT operators. As in any vector space the choice of basis is not unique: the operators are provided in the so-called Warsaw basis, which is both complete and non-redundant. The upper table contains the operators with at least one bosonic field, with φ indicating the Higgs doublet. The lower table contains the operators constructed from four fermion fields, where q and l indicate SU(2)$_L$ left-handed quark and lepton doublets and e, u, and d denote right-handed charged fermions and quarks, singlet under hypercharge. In this classification p, r, s, t denote generation indices (so they run from 1 to 3). Whenever the covariant derivate appears, it indicates the full SM covariant derivative.

As indicated by Table , at mass-dimension 6, we have basically three families of operators present in the SMEFT Lagrangian:

- *Four-fermion operators*: these are operators that contain four fermion fields, such as two lepton and two quark. One representative of this class is

$$\mathcal{O}_{qq}^{1(ijkl)} = \left(\bar{q}_i\gamma^\mu q_j\right)\left(\bar{q}_k\gamma_\mu q_l\right) , \tag{3.45}$$

  where q_i denotes a quark spinor field and i, j, k, l denote fermion generation indices. You can check that $\mathcal{O}_{qq}^{1(ijkl)}$ is a *bona fide* SMEFT operator because

  (a) It has the appropriate mass dimension: $\left[\mathcal{O}_{qq}^{1(ijkl)}\right] = 4\times[q_j] = 4\times 3/2 = 6$.

  (b) It is composed only by SM fields, in this case quark fields.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi \widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi \widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi \widetilde W B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Figure 3.4: Summary of dimension-6 SMEFT operators in the Warsaw basis.

(c) It satisfies the SM symmetries, in particular Lorentz invariance (since the operator is a Lorentz scalar) and gauge invariance, for example under a SU(3) color transformation the quark spinor transforms as

$$\psi_i^{(f)} \to U_{ij}(x)\psi_j^{(f)} = \exp\left(i\theta^a(x)t^a_{ij}\right)\psi_j^{(f)}\,, \tag{3.46}$$

which leaves the operator invariant, and where i, j are color indices, f is a flavour index, and t^a are the SU(3) generator matrices.

You can convince yourselves that there exist many other possible combinations of quark fields that satisfy the same condition, such that

$$\mathcal{O}_{qq}^{3(ijkl)} = \left(\bar{q}_i\gamma^\mu\tau^I q_j\right)\left(\bar{q}_k\gamma_\mu\tau^I q_l\right)\,, \tag{3.47}$$

which only differs from the previous operator by the presence of the SU(2) generators (the Pauli matrices) that rotate the quark doublets.

Four-fermion operators

In the absence of further assumptions, the number of four-fermion operators in the SMEFT grows very large, as you can see from all possible combinations of the $ijkl$ indices that operators such as $\mathcal{O}_{qq}^{1(ijkl)}$ and $\mathcal{O}_{qq}^{3(ijkl)}$ can exhibit. Each of these combinations is in principle independent and has associated a separate Wilson coefficient.
The number of four-fermion operators in the SMEFT can be tamed by imposing additional flavour assumptions, such as Minimal Flavour Violation [4], that assumes that the flavour structure of the UV-complete theory mirrors that of the SM.

- Operators that involve a fermion pair plus gauge fields. One example of this category is the quark Chromo-magnetic operator, defined by

$$\mathcal{O}_{uG} = \left(\bar{q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{\varphi}G^A_{\mu\nu}\,, \tag{3.48}$$

which couples the gluon field strength to the Higgs doublet, a right-handed up-type quark and a left-handed quark doublet.

As in the case of the four-fermion operators, in principle each combination of the flavour generation indices p and r gives rise to a different independent operator.

- Operators constructed entirely from gauge and Higgs fields, which are also known as *purely bosonic operators*. Two important examples of this category are

$$\mathcal{O}_\varphi = \left(\varphi^\dagger\varphi\right)^3\,, \tag{3.49}$$

which is responsible in particular for the modifications to the Higgs self-coupling, and the pure-gluonic operator

$$\mathcal{O}_G = f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho\,, \tag{3.50}$$

which modifies the SM three-gluon and four-gluon vertices and thus can affect for example multijet production at the LHC.

Go note that the operator defined in Eq. (3.50) modifies multi-jet production, note that the SU(3) gauge field strength tensor is given by

$$G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g_s f_{ABC} A^B_\mu A^C_\nu \,, \tag{3.51}$$

so therefore the product of three of such tensors will not only modify the g^3 and g^4 couplings, but also create novel ones, in particular $\mathcal{O}_G$ induces tree-level five- and six-gluon vertices which are absent in the Standard Model.

As in the case of the four-fermion operator, we can go through the usual checklist to verify that $\mathcal{O}_G$ fulfills all the conditions required to be a SMEFT dimension-six operator:

(a) The mass dimension is $[\mathcal{O}_G] = 3 \times [\partial A] = 3 \times 2 = 6$.

(b) It is composed only by SM fields, in this case the gluon.

(c) It satisfies the symmetries of the SM, in this case SU(3) color invariance.
To demonstrate the latter property, one can use the fact that the gluon field strength transforms as

$$t^A F^A_{\mu\nu} \to U(x) t^A F^A_{\mu\nu} U^{-1}(x) \,, \qquad U(x) = \exp\left(i\theta^B(x) t^B\right) \tag{3.52}$$

and use properties of the Gell-Man algebra, such as the cyclic properties of the trace.

Let's us provide some more details about the effects that some of these operators induce at the level of cross-sections and decay widths. We can consider again the quark chromo-magnetic operator

$$\mathcal{O}^{pr}_{uG} = \left(\bar{q}_p \sigma^{\mu\nu} T^A u_r\right) \widetilde{\varphi} G^A_{\mu\nu} \,, \tag{3.53}$$

which connects a right-handed quark doublet and a left-handed quark singlet field with the Higgs doublet and the gluon field strength. Further, we assume now that $p = r = 3$, that is, that the quark fields belong to the third generation. This means that $u_r = t$ (right-handed singlet) and that $\bar{q}_3$ is the top-bottom SU(2) doublet. Ignoring Lorentz and color indices, in this case this operator reads

$$\mathcal{O}^{33}_{uG} \propto \left(\bar{q}_3 \sigma^{\mu\nu} t\right) \widetilde{\varphi} G_{\mu\nu} \,. \tag{3.54}$$

To assess the kind of effects that this operator will induce, we need to break electroweak symmetry breaking writing the Higgs doublet as

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \,, \tag{3.55}$$

from where we see that $\mathcal{O}^{33}_{uG}$ contains the following interaction terms

$$\mathcal{O}^{33}_{uG} \supset v\,(\bar{t}t)\,\sigma^{\mu\nu} A_\mu A_\nu + h\,(\bar{t}t)\,\sigma^{\mu\nu} \partial_\nu A_\mu + \ldots \,. \tag{3.56}$$

The first term induces an interaction, $\propto \bar{t}tA^2$, between two gluons and two top quarks, which is absent in the SM. As mentioned above, this correction will affect top quark pair production. In Fig. 3.5 we show we show how top-quark pair production is modified by the presence of a new $ggt\bar{t}$ vertex arising from a $d = 6$ operator.

Then the second term in Eq. (3.56) modifies the interaction between two top quarks, one Higgs boson, and one gluon, and is thus relevant for the calculation of the p_T spectrum

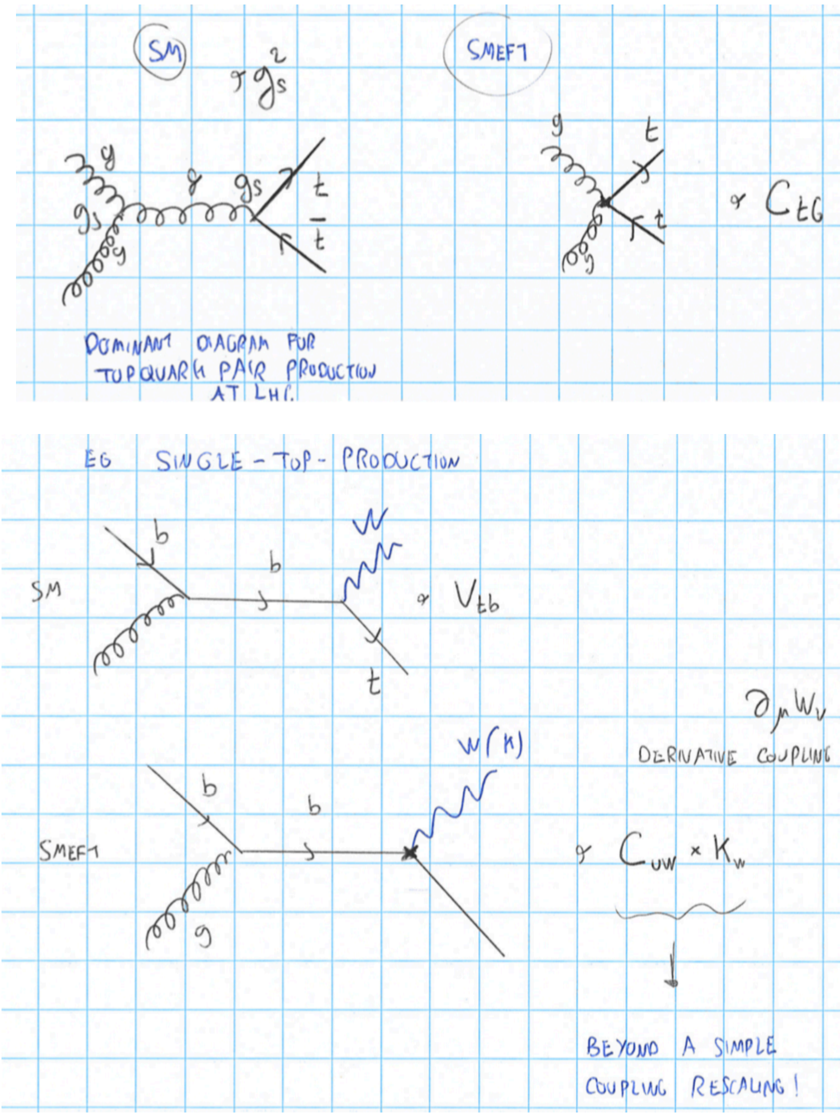


Figure 3.5: Two examples of SMEFT corrections to SM processes. In the upper case, we show how top-quark pair production is modified by the presence of a new $ggt\bar{t}$ vertex arising from a $d = 6$ operator. In the lower case, t-channel single top quark production is modified due to a new derivative coupling affecting the btW vertex.

of Higgs bosons via the gluon-fusion mechanism or the production of a Higgs boson in association with jets. See Fig. 3.6 for the corresponding Feynman diagram. This example illustrates another important property of the SMEFT: in general a single operator will contribute in a correlated way to a number of in principle independent processes, in this case top quark pair production and Higgs production in gluon fusion.

Thus if we denote by c_{tG} the Wilson coefficient associated to $\mathcal{O}_{uG}^{33}$, we have that the cross-section for processes such as $gg \to t\bar{t}$ of Higgs production in gluon fusion will be modified as

$$\sigma_{\rm tot} = \sigma_{\rm SM} \times \left(1 + \kappa \frac{c_{tG}}{\Lambda^2} + \widetilde{\kappa} \frac{c_{tG}^2}{\Lambda^4} \right) , \quad (3.57)$$

where the Wilson coefficient c_{tG} is either determined from a UV-complete theory via matching or constrained from data, the κ and $\widetilde{\kappa}$ process-dependent coefficients can be evaluated in perturbation theory, and where $\sigma_{\rm SM}$ represents the SM prediction.

As a second example, we can consider another SMEFT operator belonging to the *mixed*

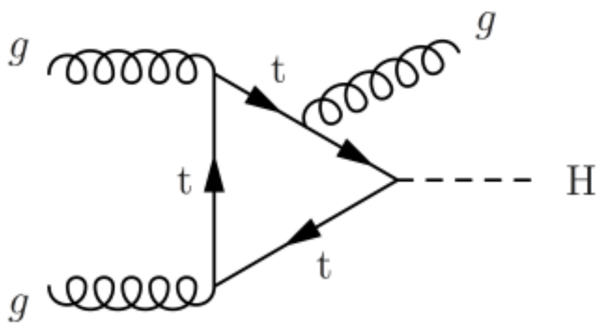


Figure 3.6: The radiation of a hard gluon modifies the p_T spectrum of Higgs bosons produced in gluon fusion as well as the cross-section of Higgs+jet production. As discussed in the text, the SMEFT dimension-six operator $\mathcal{O}_{uG}^{33}$ induces a new local interaction term $\propto h\,(\bar{t}t)\,\sigma^{\mu\nu}\partial_\nu A_\mu$ that modifies this process.

category that contains both fermionic and bosonic fields,

$$\mathcal{O}_{uW}^{pr} = \left(\bar{q}_p \sigma^{\mu\nu}\tau^I u_r\right)\widetilde{\varphi} W_{\mu\nu}^I\,, \tag{3.58}$$

and which is the analog of the chromo-magnetic operator but now with the weak field strength $W_{\mu\nu}^I$ (and therefore with the SU(2) generators τ^I). Again considering third-generation quark fields, this operator after EWSB will contain interaction terms of the form

$$\mathcal{O}_{uW}^{pr} \supset v\left(\bar{b}\sigma^{\mu\nu}t\right)\left(\partial_\mu W_\nu\right)\,, \tag{3.59}$$

which implies that this operator will modify the Wtb coupling, relevant for example for t-channel single top production or for the $t \to Wb$ decay mode of top quarks. In the lower panel of Fig. 3.6 we highlight how t-channel single top quark production is modified due to a new derivative coupling affecting the btW vertex.

An interesting feature of Eq. (3.59) is that even if in this case the SM already has a tbW coupling, it is of the from

$$\mathcal{L}_{\rm SM} \supset A(\bar{b}\gamma^\mu t)W_\mu\,, \tag{3.60}$$

while the SMEFT contribution reads instead

$$\mathcal{L}_{\rm SMEFT} \supset B\frac{v}{\Lambda^2}(\bar{b}\sigma^{\mu\nu}t)\partial_\mu W_\nu\,, \tag{3.61}$$

which is a *derivative coupling*, and where the corresponding Feynman rule will feature momentum dependent terms. Therefore the net effect of accounting for the dimension-six operator Eq. (3.58) is not just rescaling the SM tbW coupling, but actually modifying also its energy and kinematic dependence. This kind of energy-dependent effects are very important since they enhance in general the sensitivity to SMEFT effects as compared to simple coupling rescaling.

These two examples highlight some of the general properties associated to the SMEFT-induced corrections to SM processes:

SMEFT effects in SM processes

The higher-dimensional SMEFT operator can modify SM processes in two different ways. **(i)** By means of altogether *new interaction vertices*, such as the $ggt\bar{t}$ interaction that corrects top-quark pair production. **(ii)** Due to the presence of *anomalous couplings*, where existing SM interactions are modified.

Note that in the latter case in general this correction goes beyond a mere *rescaling of the coupling* (where $g_{\rm BSM} = g_{\rm SM} \times f$), for example we will have derivative interactions that induce new energy-dependent terms in the interaction vertex and thus strongly modify the kinematics of the underlying process.

3.4 Lepton and baryon number violation

Until now in our discussion above we have focused on dimension-six operators. The reader might ask why we have ignored the dimension-five operators, which from power-counting arguments would appear to be more phenomenologically relevant (due to their smaller suppression). The reason for this is that at dimension 5 one can only construct a single operator out of the SM fields, the so-called *Weinberg operator*. This operator is defined by

$$\mathcal{L}^{(d5)}_{\rm SMEFT} = c_W \frac{\left(\bar{l}_p l_p\right)\left(\varphi^\dagger \varphi\right)}{\Lambda}\,, \tag{3.62}$$

where l_p is a left-handed lepton $\text{SU}(2)_L$ doublet and φ is the Higgs doublet before electroweak symmetry breaking.

Since the l_p lepton doublet contains both left-handed charged leptons and neutrinos, one can see how after electroweak symmetry breaking, once the Higgs doublet acquires a vacuum expectation value, a fermionic mass term for the neutrinos will be generated

$$\mathcal{L}^{(d5)}_{\rm SMEFT} \supset \frac{c_W}{\Lambda}\bar{\nu}_p \nu_p v^2/2 = m_{\nu,p}\bar{\nu}_p\nu_p/2\,, \qquad m_{\nu,p} \propto \frac{c_W v^2}{\Lambda}\,. \tag{3.63}$$

Recall that in the Standard Model neutrinos are massless, though experimentally we know that they have a non-zero mass and in particular that neutrinos of different flavours can oscillate among them. Such a term violated lepton flavour violation, which is one of the accidental symmetries of the SM: as discussed in the previous section, an EFT needs to inherent the fundamental symmetries of the full theory but not the accidental ones. We should also mention that this is not the only way in which neutrinos can acquire mass, and ongoing experiments plan to establish whether the Weinberg operator Eq. (3.62) is responsible or not for the neutrino mass generation mechanism.

Furthermore, it is possible to see that the heavy mass scale responsible for Λ might be different from that associated to the dimension-six operators. If neutrinos acquire mass via this mechanism, then since experimental data tells us that $m_\nu \sim 0.05$ eV that one can estimate that the New Physics scale associated to these effects is $\Lambda \gtrsim 10^{15}$ GeV, much higher than the mass scales accessible directly or indirectly at the LHC or other experiments. This is the reason why lepton number violating operators are not relevant for collider physics, since they would arise from heavy new physics at mass scales much higher than what can be probed in these experiments. In the following, we will assume lepton number conservation and thus set to zero the Wilson coefficients associated to lepton number violating operators.

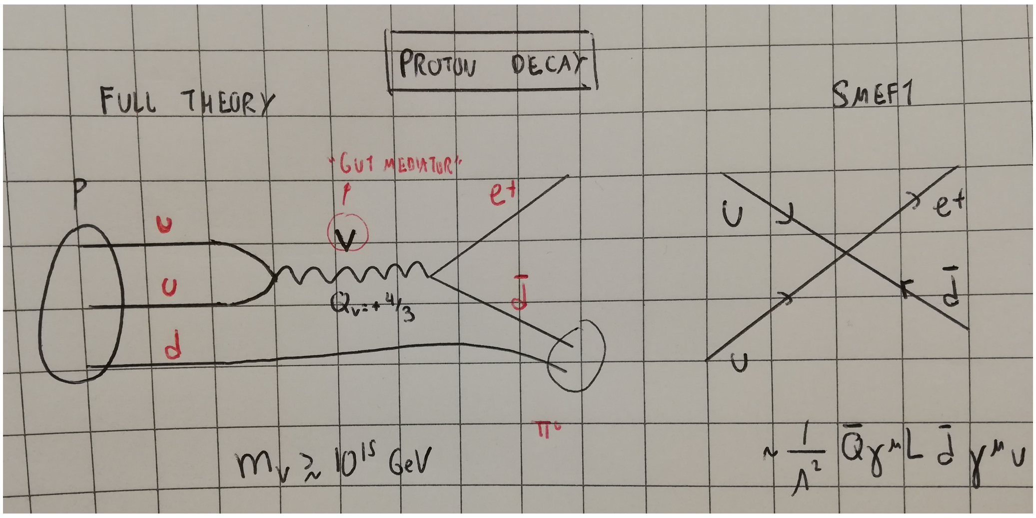


Figure 3.7: Certain $d = 6$ SMEFT operators can induce processes that violate the conservation of baryon number. In this case we see that how the operator in Eq. (3.64) leads to a $uu \to e^+\bar{d}$ transition that results into proton decay via $p \to e^+ + \pi^0$. Current bounds on the proton lifetime imply that the mass scale associated with these effects is very large, $\Lambda \gtrsim 10^{15}$ GeV.

Accidental vs fundamental symmetries

QFTs are defined by a number of *fundamental symmetries*. In the case of the SM these include gauge and Lorentz invariance and CPT invariance. Such fundamental symmetries cannot be removed without completely changing the QFT, and thus need to be satisfied also by the corresponding EFTs (else the matching procedure would be impossible).

QFTs may also exhibit *accidental symmetries*, such as conservation laws related to the specific field content of the theory rather than some deep invariance principle. In the SM lepton and baryon number conservation are accidental symmetries: they would be absent if for example the field content or the gauge charge assignments would be different (keeping all its symmetries unaffected). Accidental symmetries do not need to be necessarily satisfied by the associated EFT.

In the same way as the Weinberg operator violates lepton number conservation L, at dimension-six one can construct SMEFT operators that induce a violation of the baryon number. A representative example of a baryon-number violating operator would be

$$\mathcal{L}_{\rm SMEFT} \supset c_B \frac{1}{\Lambda^2} \bar{Q}\gamma^\mu L \bar{d}\gamma_\mu u \,, \tag{3.64}$$

which is invariant under all the symmetries of the SM but leads to processes for which $\Delta B \neq 0$. Such types of $\Delta B \neq 0$ process will in general result in proton decays. As shown in Fig. 3.7, in this case we see that how the operator in Eq. (3.64) leads to a $uu \to e^+\bar{d}$ transition that results into proton decay via $p \to e^+ + \pi^0$. Using current bounds on the proton lifetime, $\tau_p \gtrsim 10^{33}$ years, also for the mass-scale of new physics associated to this kind of baryon-number violating operators one finds stringent bounds $\Lambda \gtrsim 10^{15}$ GeV, of the same order of those associated to the Weinberg operator and that are typically related to grand-unified theories (GUTs).

Flavour structure. In the previous section we have discussed a representative number of dimension-six SMEFT operators. The complete list in the Warsaw basis was summarised in Fig. 3.4. The operators involving fermion fields have associated flavour generation indices, and in general one can construct a large number of a priori independent combinations. In the absence of additional flavour assumptions, one ends up with $N_{d6} = 2499$ independent dimension-six operators that satisfy both lepton and flavour number conservation. To reduce the complexity of these large parameter space one can introduce additional assumptions on the flavour structure of the UV-complete theory, following indications from experimental data.

One example is provided by the so-called flavor-changing neutral currents (FCNC). These are processes that involve a variation in flavour quantum numbers via the exchange of an electric-charge neutral current. One example would be the process

$$d + \bar{s} \to \bar{d} + s \,. \tag{3.65}$$

As shown in Fig. 3.8, in the SM tree-level processes with neutral currents (as the exchange of a photon or a Z boson) conserve the flavour numbers, so only for suppressed loop-induced processes such as in kaon oscillations one can have FCNC effects. This means that generically any FCNC will be extremely suppressed in the SM. However, this is another example of an *accidental symmetry* of the theory arising from its field content: there is no fundamental invariance principle that requires FCFCs to vanish in the SM, as highlighted by the fact that generically they arise at the one-loop level.

In the same way as what happens for lepton and baryon number conservation, in general the higher-dimensional operators will generate tree-level flavour-changing neutral current processes. For example, one has the operator

$$\mathcal{L}_{\rm smeft} \supset \frac{c}{M_{\rm fcnf}} \left(\bar{Q}_L^{(i)}\gamma^\mu Q_L^{(j)}\right)\left(\bar{Q}_L^{(i)}\gamma_\mu Q_L^{(j)}\right) \tag{3.66}$$

where $i \neq j$ are flavour generation indices. For $i = 1$ and $j = 2$ this operator will generate tree-level FCNCs as in Eq. (3.65), see also Fig. 3.8. Given than in the SM FCNCs are heavily suppressed, and that experimentally measurements of such processes agree with the SM predictions, allows us to impose severe bounds on the mass scale associated to these processes. Using current data, one can estimate that $M_{\rm fcnf} \gtrsim 10^6$ GeV. For this reason, in many SMEFT analysis one assumes the so-called *Minimal Flavour Violation* (MFV), which basically states that the UV-completion of the SM (and thus the SMEFT as well) satisfies the same accidental flavour symmetries as in the case of the SM.

Similar considerations arise from CP violation. The SM contains only two sources of CP violation, through the CKM and PMS fermion mixing matrices for quarks and neutrinos, and via the QCD θ term. These sources of CP violation are highly suppressed for most processes, for example the neutron and electron dipole moments are vanishingly small in the SM. However SMEFT operators can generically induce CP-violating processes and thus a non-zero neutron and electron dipole moments. An example of such CP-violating operator would be

$$\frac{c_{\rm CPV}}{\Lambda_{\rm CPV}} \left(\bar{q}_p \sigma^{\mu\nu} u_n\right) \widetilde{\varphi} B_{\mu\nu} \,, \tag{3.67}$$

The fact that so far we have only obtained upper bounds on the neutron and electron dipole moments implies that such effects need to be very small, so available data indicates that the mass scale associated to CPV effects will be large, $\Lambda_{\rm CPV} \gtrsim 10^7$ GeV. Therefore for many SMEFT analyses it is reasonable to assume that CP is conserved and thus that we can set to zero the Wilson coefficients associated to CP-violating operators.

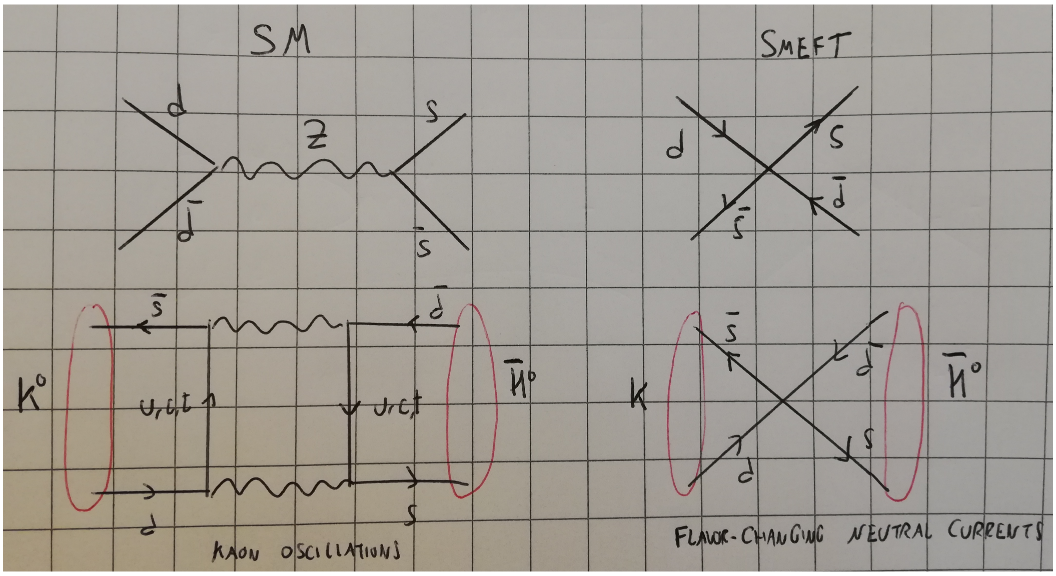


Figure 3.8: Dimension-6 operators in the SMEFT can induce large flavour-changing neutral current effects, which are severely constrained by experimental data. In the SM these FCNC transitions can only arise at loop-level, such as in the case of kaon oscillations, and thus are generically suppressed.

The SMEFT at the LHC

With these two generic assumptions, Minimal Flavor Violation and absence of CP-violation, the number of dimension-six SMEFT operators relevant of the interpretation of LHC measurements becomes of order 60. These number can be increased in a controlled way as the assumptions related to the accidental symmetries of the SM UV-completion are relaxed.

Following this walk-through about some representative SMEFT operators, we can present one explicit example of matching in the SMEFT from a UV complete theory. As we will see, from the conceptual point of view this matching mechanism will be not too dissimilar from that presented in Sect. 2.

3.5 Matching in the SMEFT: lepton-proton scattering

We would like now to present an explicit example of a calculation of a Standard Model process that is modified from the presence of the SMEFT effects arising from some UV-complete theory. This case study will be lepton-proton scattering when the Standard Model is extended with a heavy gauge boson Z' with mass $m_{Z'} \gg m_Z$.

Let us first review the main features of this process, known as deep-inelastic scattering. In this process, a high-energy lepton scatters off one of the quarks in the proton by means of the exchange of a virtual photon, as indicated in Fig. 3.9, namely

$$e^-(k) + p(p) \to e^-(k') + X \, , \tag{3.68}$$

where X denotes the hadronic final state of the collision. The four momentum transfer between the lepton and the proton is given by

$$q \equiv k' - k \, . \tag{3.69}$$

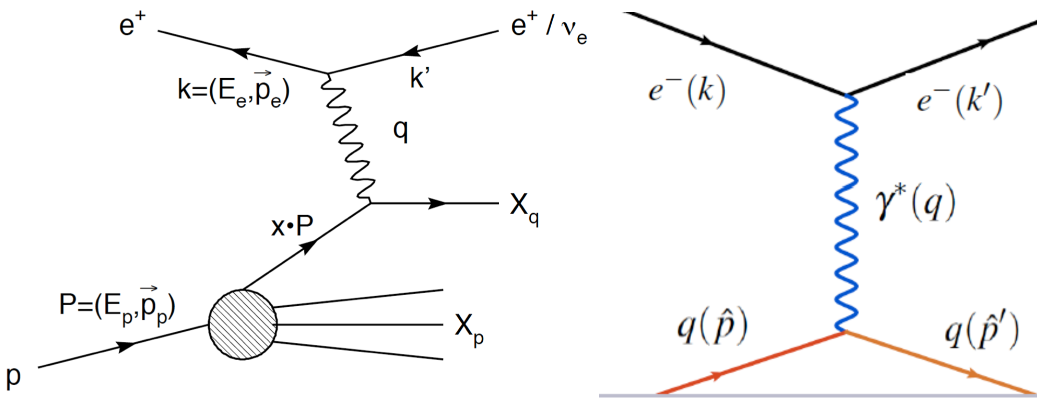


Figure 3.9: Left plot: the deep-inelastic lepton-proton scattering process. Right plot: the Born contribution to the lepton-quark scattering.

The kinematics of the deep-inelastic scattering process are specified by the following variables

$$x \equiv \frac{Q^2}{2\,p\cdot q}, \quad Q^2 \equiv -q^2\,, \quad y \equiv \frac{q\cdot p}{k\cdot p}\,. \tag{3.70}$$

where $Q^2 \gg M_p^2$ ensures the validity of the perturbative QCD description. The center-of-mass energy of the proton-virtual photon collision will be

$$W^2 \equiv (p+q)^2 = M_p^2 + Q^2\frac{1-x}{x} \simeq Q^2\frac{1-x}{x}\,, \tag{3.71}$$

where the proton mass can be neglected in the calculation. The value $x = 1$ is known as the elastic limit. An important property of DIS is that the complete kinematics of the process are fully specified by measuring the four-momenta of the outgoing lepton. This process was measurement with unprecedented precision at the HERA lepton-proton collider, that operated in DESY Hamburg between 1992 and 2007.

Restricting ourselves to virtual-photon exchange, the DIS lepton-proton cross-section can be written in terms of a *structure function* $F_2(x)$. In turn this structure function is a convolution between the $\gamma^* q \to X$ partonic cross-section and the PDFs of the proton,

$$\frac{Q^4 x}{2\pi\alpha^2_{\rm QED}\left(1+(1-y)^2\right)}\frac{d^2\sigma^{\rm DIS}}{dxdQ^2} = F_2(x) = \sum_{\rm q,\bar{q}}\int_x^1 \frac{dz}{z} f_q(z)\widehat{\sigma}_{q\gamma^*\to X}\left(\frac{x}{z}\right)\,. \tag{3.72}$$

The proton PDF $f_q(z)$ indicates the likelihood of finding quark q in the proton carrying a fraction z of its longitudinal momentum. For example $f_u(z = 0.6)$ measures the probability of finding an up quark in the proton carrying 60% of its energy. These PDFs cannot be computed from first principles (unlike the partonic cross-section $\widehat{\sigma}$ which can be evaluated in perturbation theory) and must therefore be evaluated from the experiment by means of the so-called global QCD fits. Furthermore, at high-energies additional contributions need to be added to Eq. (3.72) to account for the fact that the incoming lepton can also interact with the proton target by means of the exchange of a Z boson.

Now let us assume that the Standard Model can be expected with a new heavy gauge boson Z', which couples in a non-universal way to quarks and charged leptons. In this case the full theory Lagrangian will be the SM Lagrangian supplemented with the interaction terms between the new gauge boson and the SM fermions, namely

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + g_{l,f}\bar{l}_f\gamma^\mu(1+\gamma^5)l_f Z'_\mu + g_{q,f'}\bar{q}_{f'}\gamma^\mu(1+\gamma^5)q_{f'}Z'_\mu + \ldots\,, \tag{3.73}$$

where l_f and $q_{f'}$ indicate the fermion fields associated to the charged leptons and quarks with flavour f and f' respectively, and the strength of their couplings to the Z' boson is dictated by the $g_{l,f}$ and $g_{q,f'}$ coefficients. Note that in this model the interactions of the Z' boson are non-universal: for example, it can couple with different strengths to electrons, moons, and tau leptons. The dots in Eq. (3.73) indicate the kinetic terms, with possible mixing terms with A_μ and Z_μ. The mass of the Z' gauge boson, $M_{Z'}$, is assumed to be much higher than the electroweak scale.

We now need to take the low energy limit of this UV-complete theory, $E \ll M_{Z'}$, and construct the effective Lagrangian that arises once the heavy Z' boson has been integrated out. We know, by construction, that the operators that we will obtain will be a subset of the dimension-6 SMEFT operators. The main advantage of this top-down approach is the knowledge of the UV-complete theory allows us to:

- Be able to identify which of the 2499 dimension-6 SMEFT operators are non-zero for this specific UV completion.
- Determine the values of the corresponding Wilson coefficients from matching with the UV complete theory, via amplitude matching or with some other method.

Note that the second point will allow us to express the Wilson coefficients in terms of the mass and couplings of the new heavy Z' boson. If the UV completion was unknown, we would need to set bounds on the Wilson coefficients from experimental measurements instead.

In the Warsaw basis of the SMEFT, one notes that the following operators are present:

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}} &\supset \frac{c_{f,f'}}{\Lambda^2}\left(\bar{l}_{f,R}\gamma^\mu l_{f,R}\right)\left(\bar{q}^f_R\gamma_\mu q^f_R\right) \\ &= \frac{c_{f,f'}}{16\Lambda^2}\,\bar{l}_f(1-\gamma^5)\gamma^\mu(1+\gamma^5)l_f\;\bar{q}_{f'}(1-\gamma^5)\gamma_\mu(1+\gamma^5)q_{f'}\,, \end{aligned} \tag{3.74}$$

where $l_{f,R}$ and $q_{f',R}$ are respectively the right-handed fermion fields for charged leptons and quarks, Λ is the generic high-scale that determines the validity of the EFT description, and where we have used the fact that $P_R = (1+\gamma_5)/2$ projects fermionic fields into their right-handed components. These operators belong to the class of four-fermion operators, and the Wilson coefficients $c_{f,f'}$ are in principle flavor non-universal, that is, for every charged lepton and quark generation they can be different. This expression can be simplified by means of Dirac algebra Using Dirac algebra, $(1-\gamma^5)\gamma^\mu(1+\gamma^5) = \gamma^\mu(2+2\gamma^5)$ so that one can express these four-fermion operator as follows:

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{c_{f,f'}}{4\Lambda^2}\bar{l}_f\gamma^\mu(1+\gamma^5)l_{f'}\;\bar{q}_{f'}\gamma_\mu(1+\gamma^5)q_{f'}\,. \tag{3.75}$$

It is possible to show that four-fermion operators of the form of Eq. 3.75 can be generated from the low-energy limit of a UV complete theory as that encoded by Eq. (3.73). Indeed, a UV complete theory with a Z' boson with mass $M_{Z'}$ that couples to the SM charged leptons and the quarks by means of the interaction term

$$\mathcal{L}_{\text{full}} \supset g_{l,f}\bar{l}_f\gamma^\mu(1+\gamma^5)l_f Z'_\mu + g_{q,f'}\bar{q}_{f'}\gamma^\mu(1+\gamma^5)q_{f'}Z'_\mu\,, \tag{3.76}$$

with lead to the sought-for four-fermion operators by means of tree-level matching conditions. In particular, we can consider matching via the following scattering amplitude

$$\mathcal{M}\left(l_f\,+\,q_{f'}\,\to l_f\,+\,q_{f'}\right)\,, \tag{3.77}$$

in which a high energy charged lepton l_f, for example an electron or a positron, scatters of a quark $q_{f'}$, for example an up or down quark. In Fig. 3.10 we represent the matching

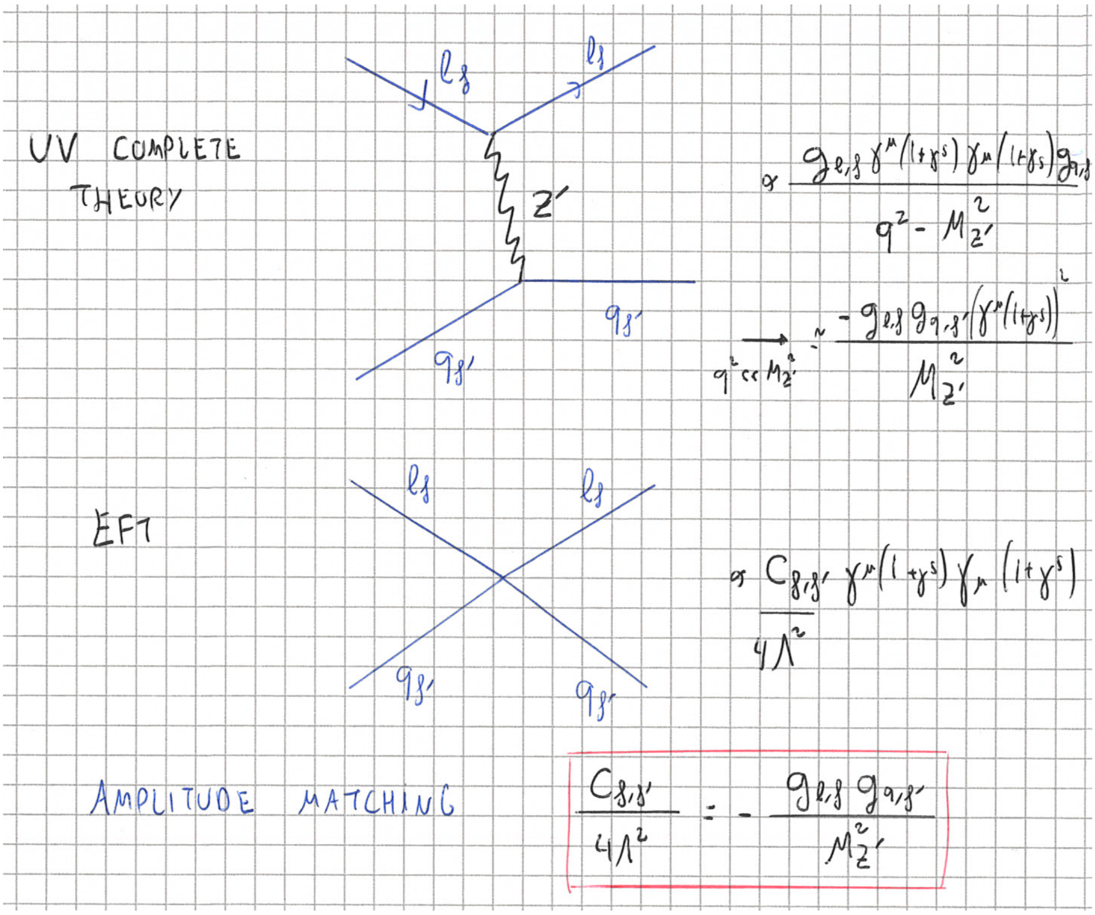


Figure 3.10: By means of tree-level matching, we can relate the Wilson coefficients of the dimension-six SMEFT operators to the masses and couplings of the heavy gauge Z' boson present in the full (UV-complete) theory.

between the full and the effective theories by means of this specific scattering amplitude. It should be clear that when we take the $q^2 \ll M_{Z'}$ limit, in which the EFT description is valid, we can equate the amplitudes in the full and the effective theory and express the Wilson coefficients in the EFT in terms of the masses and couplings of the full theory.

So in summary we have shown that the four-fermion operators of Eq. (3.75) can be understood as the low-energy limit of a renormalizable, UV-complete Lagrangian where a new heavy gauge boson Z' couples in a non-universal way to quarks and charged leptons. By means of tree-level matching, we can relate the Wilson coefficients of the dimension-six SMEFT operators to the masses and couplings of the heavy gauge Z' boson present in the full (UV-complete) theory, finding

$$-\frac{g_{l,f}g_{q,f'}}{M_{Z'}^2} = \frac{c_{f,f'}}{4\Lambda^2} \,. \tag{3.78}$$

Thanks to this calculation, we can identify the generic cut-off scale of the EFT with the mass of the Z' boson and we have derived a relation between the SMEFT Wilson coefficients and the Z' boson coupling constants

$$\Lambda = M_Z \,, \qquad c_{f,f'} = -4g_{l,f}g_{q,f'} \,. \tag{3.79}$$

Note that in general we will have one different four-fermion operator for each combination of lepton f and quark f' flavour. This is why four-fermion operators dominate the count of

2499 dimension-6 operators in the SMEFT Lagrangian: there are many ways in which we can connect their flavour indices. If the UV-complete model exhibits additional symmetries then the matching conditions simplify: for example if the Z' boson couples universally to fermion flavours (as happens in the Standard Model) then we will have a single dimension-6 operator contribution to the cross-section with the associated Wilson coefficient now given by

$$\Lambda = M_Z \,, \qquad c = -4 g_l g_q \,. \tag{3.80}$$

Next we want to evaluate how the SMEFT operators (now that we have matched them to the couplings and masses of the UV-complete cross-sections) affect the SM theory predictions. The advantage of this formalism is that it allows us to compute how the effects of the four-fermion operators modify the DIS structure functions by analogy to the corresponding calculation in the SM. As mentioned above, experimental measurements of inclusive neutral-current deep-inelastic are expressed in terms of structure functions:

$$\frac{d^2\sigma^{\mathrm{NC},l^\pm}}{dx dQ^2}(x,Q^2) = \frac{2\pi\alpha^2}{xyQ^4}\left[Y_+\, F_2^{\mathrm{NC}}(x,Q^2) \mp Y_-\, xF_3^{\mathrm{NC}}(x,Q^2) - y^2\, F_L^{\mathrm{NC}}(x,Q^2)\ \right] \tag{3.81}$$

where $Y_\pm = 1 \pm (1-y)^2$ and the DIS kinematic variables are given in Lorentz-invariant form as

$$x = \frac{Q^2}{2P\cdot q}\,, \qquad Q^2 = -q^2\,, \qquad y = \frac{q\cdot P}{k\cdot P}\,, \tag{3.82}$$

in terms of the four-momenta of the proton, the charged lepton, and the exchanged virtual boson (γ^* or Z) P, k, and q, respectively. At leading order, the Bjorken variable x can be interpreted as the fraction of the nucleon's momentum carried by the struck parton. The virtuality of the exchanged gauge boson, Q^2, represents the hardness of the scattering reaction.

In the following we evaluate the leading-order SMEFT corrections to the structure functions $F_2^{\mathrm{NC}}(x,Q^2)$, those associated to F_3 and F_L can be evaluated in a fully equivalent way. We will need to evaluate three terms. Recall that in the Standard Model the scattering amplitude $\mathcal{M}\left(l_f + q_{f'} \to l_f + q_{f'}\right)$ proceeds either via the exchange of a virtual photon γ^* or of a Z boson, via similar diagrams as that illustrated in Fig. 3.10. First of all, the interference between the amplitude involving a Z' boson exchange, determined by the Lagrangian Eq. (3.73), and denoted by $\mathcal{M}_{Z'}$. This will lead to two new contributions to the F_2 structure function, one arising from the $\mathcal{M}_Z\mathcal{M}_{Z'}$ interference and the other from the $\mathcal{M}_\gamma\mathcal{M}_{Z'}$ interference, where $\mathcal{M}_\gamma$ and $\mathcal{M}_Z$ denote the SM amplitudes for virtual photon and Z boson exchange respectively. Then, we will need to compute the square of the amplitude involving a Z' boson exchange, $|\mathcal{M}_{Z'}|^2$. Note that by convention, the photon propagator enters the differential cross section as a $1/Q^4$ prefactor for all the structure functions, so to cancel this in the definition of F_2 and F_3, we include it in the numerator.

We will assume for simplicity that the Z' boson couples universally to leptons, with coupling g_l, and non-universally to quarks with flavour q_f, with coupling g_f. The contribution from the $|\mathcal{M}_{Z'}|^2$ term to F_2 has the same form as that of $\mathcal{M}_Z\mathcal{M}_Z$ in the SM, namely

$$F_2 \supset \frac{Q^4}{(q^2 - M_{Z'}^2)^2}\ \left(V_{qZ'}^2 + A_{qZ'}^2\right)\left(V_{lZ'}^2 + A_{lZ'}^2\right)\left(q(x,Q^2) + \bar{q}(x,Q^2)\right)\,, \tag{3.83}$$

where $V_{iZ'} = g_i$ and $A_{iZ'} = -g_i$ are the relevant vector and axial couplings with the Z' respectively, which gives

$$F_2 \supset \frac{4g_l^2 g_f^2 Q^4}{(Q^2 + M_{Z'}^2)^2}\ \left(q(x,Q^2) + \bar{q}(x,Q^2)\right)\,. \tag{3.84}$$

Integrating out the heavy Z' field and using Eq. (3.78) leads to:

$$F_2 \supset \frac{4Q^4 g_l^2 g_f^2}{M_{Z'}^4}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) = 4c_f^2\frac{Q^4}{4\Lambda^4}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) . \quad (3.85)$$

Concerning the term arising from the interference between the Z' and γ^* exchange, the relevant term in F_2 is, again, identical to the Z/γ^* interference term in the SM structure functions, which is given by

$$\begin{aligned} F_2 &\supset -\frac{Q^4}{q^2(q^2-M_{Z'}^2)}\left(2e^2 Q_f V_{qZ'}V_{lZ'}\right)\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &= -\frac{Q^2}{Q^2+M_{Z'}^2}\left(2e^2 Q_f g_e g_f\right)\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &= -2e^2 Q_f Q^2\frac{g_e g_f}{M_{Z'}^2}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &\longrightarrow c_f e^2 Q_f\frac{Q^2}{2\Lambda^2}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) , \end{aligned} \quad (3.86)$$

where Q_f is the fractional quark charge, $Q_u = Q_d = 2/3$ and $Q_d = Q_s = -1/3$.

There is no analog of Z'/Z interference in the Standard Model structure functions, as there was in the previous two terms. The Z' interference with Z reads:

$$\begin{aligned} F_2 &\supset \frac{2Q^4}{(q^2-M_{Z'}^2)(q^2-M_Z^2)}(V_{qZ'}V_{qZ}+A_{qZ'}A_{qZ})(V_{lZ'}V_{lZ}+A_{lZ'}A_{lZ})\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &= \frac{2g_f g_e Q^4}{(Q^2+M_{Z'}^2)(Q^2+M_Z^2)}(V_{qZ}-A_{qZ})(V_{lZ}-A_{lZ})\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &= \frac{Q^4}{Q^2+M_Z^2}\frac{2g_e g_f}{M_{Z'}^2}(-2Q_f e s_W^2)(2e s_W^2)\frac{1}{4c_W^2 s_W^2}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) \\ &\longrightarrow \frac{c_f}{2}\frac{Q^2}{Q^2+M_Z^2}\frac{Q^2}{\Lambda^2}4s_W^4 Q_f e^2\frac{1}{4s_W^2 c_W^2}\left(q(x,Q^2)+\bar{q}(x,Q^2)\right) , \end{aligned} \quad (3.87)$$

where the following vector and axial couplings between the fermions and the Z boson in the SM have been used:

$$\begin{aligned} V_q^Z &= \tfrac{1}{2c_W s_W}\left(I_3^f - 2Q_f s_W^2\right) , \qquad & V_e^Z &= -\frac{1}{2c_W s_W}\left(\frac{1}{2}-2e s_W^2\right) , \\ A_q^Z &= \tfrac{1}{2c_W s_W}I_3^f , & A_e^Z &= -\frac{1}{4c_W s_W} \end{aligned} \quad (3.88)$$

and where $s_W^2 = 1 - c_W^2 = \sin^2\theta_W$ and θ_W is the weak mixing angle.

Combining these various calculations, the expression for $F_2(x,Q^2)$ in the presence of the SMEFT dimension-6 $\mathcal{O}_{lu}$, $\mathcal{O}_{ld}$, $\mathcal{O}_{ls}$, and $\mathcal{O}_{lc}$ operators, defined by

$$\mathcal{O}_{lf} \equiv \left(\bar{l}\gamma^\mu(1+\gamma^5)l\right)\left(\bar{q}_f\gamma_\mu(1+\gamma^5)q_f\right) . \quad (3.89)$$

is given by the following result:

$$\begin{aligned} F_2(x,Q^2) &= F_2^{\rm SM}(x,Q^2) \\ &+\frac{x}{12e^4}\Bigg[\left(4c_u e^2\frac{Q^2}{\Lambda^2}(1+4K_Z s_W^4)+3c_u^2\frac{Q^4}{\Lambda^4}\right)\left(u(x,Q^2)+\bar{u}(x,Q^2)\right) \\ &\quad+\left(-2c_d e^2\frac{Q^2}{\Lambda^2}(1+4K_Z s_W^4)+3c_d^2\frac{Q^4}{\Lambda^4}\right)\left(d(x,Q^2)+\bar{d}(x,Q^2)\right)\Bigg] , \end{aligned} \quad (3.90)$$

where as before $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$, and where have have defined

$$K_Z = \frac{Q^2}{4c_W^2 s_W^2 (Q^2 + M_Z^2)} \,. \tag{3.91}$$

If expressed in terms of the parameters of the UV complete model, one has instead

$$\begin{aligned} F_2(x,Q^2) \;&=\; F_2^{\rm SM}(x,Q^2) \\ &+ \frac{x}{12e^4}\Bigg[\left(-16 g_l g_u e^2 \frac{Q^2}{M_{Z'}^2}(1+4K_Z s_W^4) + 24 g_l^2 g_u^2 \frac{Q^4}{M_{Z'}^4}\right)\left(u(x,Q^2)+\bar{u}(x,Q^2)\right) \\ &+ \left(+8 g_l g_d e^2 \frac{Q^2}{M_{Z'}^2}(1+4K_Z s_W^4) + 24 g_l^2 g_d^2 \frac{Q^4}{M_{Z'}^4}\right)\left(d(x,Q^2)+\bar{d}(x,Q^2)\right)\Bigg] \,, \end{aligned} \tag{3.92}$$

There are a number of interesting observations that one can derive from this calculation:

- In general the SMEFT corrections are *flavour-sensitive*: in this case the coupling of the Z' boson can be different between up and down quarks, and thus the same holds for the associated Wilson coefficients.

- SMEFT corrections from dimension-6 operators are of two types: $\mathcal{O}\left(\Lambda^{-2}\right)$ effects, which arise from the interference between the SM and the dimension-6 operators, and $\mathcal{O}\left(\Lambda^{-4}\right)$, which arise from the square of the SMEFT operators.

- After matching, the SMEFT corrections are written entirely in terms of the UV-complete theory parameters. So one can derive phenomenological bounds on the Wilson coefficients c_u and c_d once and for all, and these bounds will apply to *any* BSM scenario that reduces to the SM at low energies.

- We see how for this process the SMEFT corrections induce energy-growing effects, of the form Q^2/Λ^2 and Q^4/Λ^4. These effects are a consequence of the non-renormalizable nature of the SMEFT operators, and appear generically in SMEFT calculations.

  From the phenomenological point of view, this kind of effects are important since they increase the sensitivity to the c_f coefficients in the high-energy tails of experimental distributions. Of course in any case one should ensure that one remains in the regime of validity of the EFT, which in this case corresponds to the condition that

  $$Q^2 \ll \Lambda^2 \ (M_Z'^2) \,. \tag{3.93}$$

  Only in this limit the EFT description if theoretically reliable.

Next we move to another example showing how SMEFT operators modify SM processes, specifically in the case of the decays of the top quark.

3.6 Top quark decays in the SMEFT

Let us now consider a practical application of the SMEFT, namely the modifications of the decay properties of the top quark. Being the most massive quark in the SM, $m_t \simeq 173$ GeV, the top quark plays an important role in many models of new physics beyond the SM in particular with its association in the mechanism of electroweak symmetry breaking. The top quark decays almost entirely into bottom quarks, positrons, and neutrinos:

$$t \to b + l^+ + \nu_l \,. \tag{3.94}$$

Two of the operators in the SMEFT that modify the tbW coupling are given by

$$\mathcal{O}_{\phi q}^{(3)} \equiv i\left(\phi^\dagger \tau^I D_\mu \phi\right)\left(\bar{q}\gamma^\mu \tau^I q\right) , \tag{3.95}$$

$$\mathcal{O}_{tW} \equiv \left(\bar{q}\sigma^{\mu\nu}\tau^I t\right)\widetilde{\phi} W_{\mu\nu}^I . \tag{3.96}$$

Note that these operators are defined before electroweak symmetry breaking, where the W and Z boson still remain massless. In these operators ϕ is the SU(2) Higgs doublet, $\widetilde{\phi}_0 = \epsilon_{ij}\phi^{j*}$, $\tau^I/2$ represent the SU(2) generators (the Pauli matrices), t is a right-handed quark singlet and q is a left-handed quark SU(2) doublet. In the SM the full covariant derivative is given by

$$D_\mu = \partial_\mu - ig_s\frac{1}{2}\lambda^A G_\mu^A - ig\frac{1}{2}\tau^I W_\mu^I - ig' Y B_\mu . \tag{3.97}$$

In order to determine the effects that these SMEFT operators induce in the decay of the top quarks, first of all we need to evaluate them after electroweak symmetry breaking, that is, when the Higgs doublet is written as

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} . \tag{3.98}$$

Furthermore, since the decay of the top quark proceeds via the $t \to Wb$ vertex we are only interested in operators that couple the top quark with the W boson and therefore we can restrict the covariant derivative to the following:

$$D_\mu = \partial_\mu - i\frac{g}{2}\tau^I W_\mu^I . \tag{3.99}$$

Now we will derive the modifications of the $t \to Wb$ vertex due to these two effective operators. We will do one case explicitly during the lectures and the other will be left for the tutorials.

Problem II.1: The top quark decay in the SMEFT

The semi-leptonic decay of the top quark is dominated by the $t \to be^+\nu_e$ process. Two of the dimension-six operators in the SMEFT that modify the Wtb vertex are

$$\mathcal{O}_{\phi q}^{(3)} = i\left(\phi^\dagger \tau^I D_\mu \phi\right)\left(\bar{q}\gamma^\mu \tau^I q\right) , \tag{3.100}$$

$$\mathcal{O}_{tW} = \left(\bar{q}\sigma^{\mu\nu}\tau^I t\right)\widetilde{\phi} W_{\mu\nu}^I . \tag{3.101}$$

With these two operators we can write the Lagrangian of the theory as follows

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{SM} + \frac{c_{\phi q}^{(3)}}{\Lambda^2}\mathcal{O}_{\phi q}^{(3)} + \frac{c_{tW}}{\Lambda^2}\mathcal{O}_{tW} . \tag{3.102}$$

Evaluate the effective Lagrangian after electroweak symmetry breaking, and schematically derive the modifications of the Wtb induced by the new terms. Their effects are limited to anomalous couplings or we have altogether new interaction terms?

4 Phenomenology of the SMEFT

In this third part of the lectures we highlight some of the phenomenological implications of the SMEFT. First of all we discuss how the Wilson coefficients of the SMEFT operators can be constrained from experimental data. Then we illustrate the main properties of a global SMEFT analysis with some recent results related on the top quark sector. We also discuss some the phenomenological applications of the SMEFT for flavour physics and for the studies of the Higgs and electroweak sectors.

4.1 Fitting methodologies

First of all, as we discussed since in the SMEFT in general the UV-completion is assumed to be unknown we need to determine the Wilson coefficients $\{c_n\}$ associated to the higher-dimensional operators directly from experimental measurements. In order to extract the Wilson coefficients from experimental data, one can use different statistical methodologies or fitting frameworks. The starting point is to write down the theoretical expression for the i-th cross-section to be used in the fit, in terms of the SM cross-section, the Wilson coefficients c_n and the cut-off scale Λ,

$$\sigma_i^{(\rm th)}\left(\{c_n\}\right) = \sigma_{{\rm SM},i} + \sum_{n=1}^{N_{\rm op}} \widetilde{\sigma}_{i,n}\frac{c_n}{\Lambda^2} + \sum_{n,m=1}^{N_{\rm op}} \widetilde{\sigma}_{i,nm}\frac{c_n\, c_m}{\Lambda^4}\,, \qquad i=1\,\ldots,N_{\rm dat} \tag{4.1}$$

where $\widetilde{\sigma}_{i,n}$ and $\widetilde{\sigma}_{i,nm}$ are perturbative calculable cross-sections obtained from the $d=6$ SMEFT operators, and that depend on the details of the experimental measurement such as the center of mass energy, the kinematic cuts, and the specific binning chosen for the dataset. With current technology, $\widetilde{\sigma}_{i,n}$ and $\widetilde{\sigma}_{i,nm}$ can be evaluated in many cases at NLO in both the strong and electroweak coupling, using automated tools such as `MadGraph5_aMC@NLO` interfaced to the `SMEFT@NLO` model. The SM cross-section $\sigma_{{\rm SM},i}$ should be evaluated by means state-of-the-art QCD and EW calculations, for instance many LHC processes are now available at NNLO in the QCD coupling.

Next one needs to define a figure of merit to quantify the goodness of fit between the theoretical predictions expressed in terms of the Wilson coefficients, Eq. (4.1), and the corresponding experimental measurements. One possible choice of this figure of merit is given by

$$\chi^2(\{c_n\}) \equiv \frac{1}{N_{\rm dat}}\sum_{i,j=1}^{N_{\rm dat}}\left(\sigma_i^{(\rm th)}\left(\{c_n\}\right)-\sigma_i^{(\rm dat)}\right)({\rm cov}^{-1})_{ij}\left(\sigma_j^{(\rm th)}\left(\{c_n\}\right)-\sigma_j^{(\rm dat)}\right) \tag{4.2}$$

where cov indicates the total covariance matrix, that takes into account all the experimental and theoretical uncertainties (and their correlations) affecting the measurements and the associated experimental predictions. Assuming that theory and experimental uncertainties are uncorrelated, one can express the covariance matrix as

$$\text{cov}_{ij} = \text{cov}_{ij}^{(\rm exp)} + \text{cov}_{ij}^{(\rm th)} \tag{4.3}$$

where the experimental covariance matrix can be evaluated for example using the t_0 prescription

$$\begin{aligned}(\text{cov}_{t_0})_{ij}^{(\rm exp)} \;\;&\equiv\;\; \left(\sigma_i^{(\rm stat)}\right)^2\delta_{ij} + \left(\sum_{\alpha=1}^{N_{\rm sys}}\sigma_{i,\alpha}^{(\rm sys)}\sigma_{j,\alpha}^{(\rm sys)}\mathcal{O}_i^{(\rm exp)}\mathcal{O}_j^{(\rm exp)}\right.\\ &\left. + \sum_{\beta=1}^{N_{\rm norm}}\sigma_{i,\beta}^{(\rm norm)}\sigma_{j,\beta}^{(\rm norm)}\mathcal{O}_i^{(\rm th,0)}\mathcal{O}_j^{(\rm th,0)}\right)\end{aligned} \tag{4.4}$$

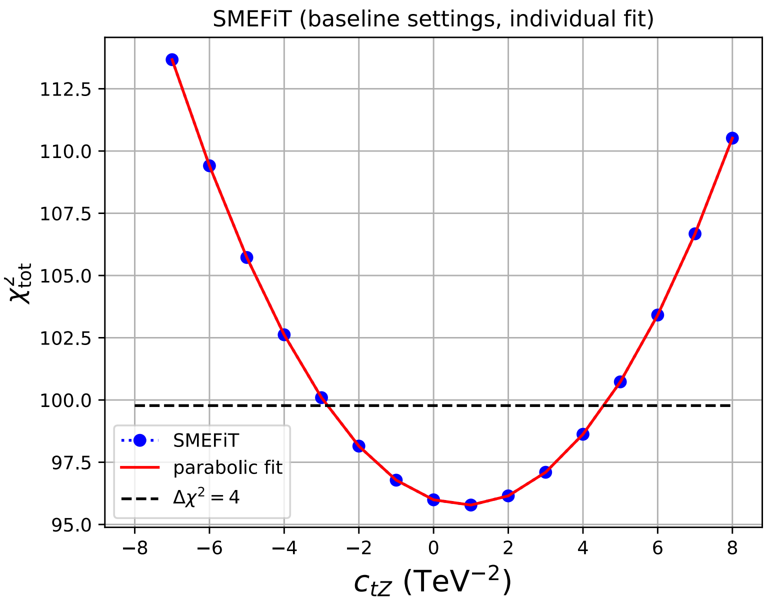


Figure 4.1: In the case of one-parameter fits, it suffices to scan the χ^2 as a function of the Wilson coefficient around the best-fit value, and determine the 95% confidence level intervals from the $\Delta\chi^2 = 4$ condition.

while for theory uncertainties one should include for example PDF uncertainties and missing higher order uncertainties (MHOUs).

The minimisation of the χ^2 Eq. (4.2) determines the best-fit values of the Wilson coefficients. However such best-fit values are not particularly useful unless they are accompanied by a robust estimate of the associated uncertainties. There exist many methods to determine those uncertainties from a SMEFT fit, as well as the correlations between different Wilson coefficients. The simplest situation corresponds to one-parameter fits, where only one coefficient is vary at a time and all the others are set to zero. For instance, we can consider the $\mathcal{O}_{tZ}$ operator, relevant for the interpretation of top quark measurements presented below. For such single parameter fit the χ^2 is given by

$$\chi^2(c_{tZ}) \equiv \frac{1}{N_{\rm dat}} \sum_{i,j=1}^{N_{\rm dat}} \left(\sigma_i^{(\rm th)}\left(c_{tZ}\right) - \sigma_i^{(\rm dat)}\right) ({\rm cov}^{-1})_{ij} \left(\sigma_j^{(\rm th)}\left(c_{tZ}\right) - \sigma_j^{(\rm dat)}\right) \qquad (4.5)$$

As shown in Fig. 4.1, in the case of one-parameter fits, it suffices to scan the χ^2 as a function of the Wilson coefficient around the best-fit value, and determine the 95% confidence level intervals from the $\Delta\chi^2 = 4$ condition. Note that in this case one should find that a quartic polynomial of the form

$$\chi^2_{\rm pol}(c_{tZ}) = \sum_{j=0}^{4} a_n {c_{tZ}}^n \,, \qquad (4.6)$$

describes the χ^2 profile, since the latter is a quartic form of the Wilson coefficients. The reason for this property is that $\sigma_j^{(\rm th)}\left(c_{tZ}\right)$ is either a linear (if only the interference term is kept) or quadratic (if also the square terms are included) form in the Wilson coefficients.

4.2 Top quark physics

In a realistic SMEFT analysis one has to combine a large number of experimental measurements from different process, and the associated theory calculations will depend on a

Notation	Sensitivity at $\mathcal{O}(\Lambda^{-2})$ ($\mathcal{O}(\Lambda^{-4})$)								
	$t\bar{t}$	single-top	tW	tZ	$t\bar{t}W$	$t\bar{t}Z$	$t\bar{t}H$	$t\bar{t}t\bar{t}$	$t\bar{t}b\bar{b}$
OQQ1								✓	✓
OQQ8								✓	✓
OQt1								✓	✓
OQt8								✓	✓
OQb1								(✓)	✓
OQb8								(✓)	✓
Ott1								✓	✓
Otb1								(✓)	✓
Otb8								✓	✓
OQtQb1									
OQtQb8									
O81qq	✓				✓	✓	✓	✓	✓
O11qq	✓				(✓)	(✓)	(✓)	✓	✓
O83qq	✓	✓		(✓)	✓	✓	✓	✓	✓
O13qq	✓	✓		✓	(✓)	(✓)	(✓)	✓	✓
O8qt	✓				✓	✓	✓	✓	✓
O1qt	✓				(✓)	(✓)	(✓)	✓	✓
O8ut	✓					✓	✓	✓	✓
O1ut	✓					(✓)	(✓)	✓	✓
O8qu	✓					✓	✓	✓	✓
O1qu	✓					(✓)	(✓)	✓	✓
O8dt	✓					✓	✓	✓	✓
O1dt	✓					(✓)	(✓)	✓	✓
O8qd	✓					✓	✓	✓	✓
O1qd	✓					(✓)	(✓)	✓	✓
OtG	✓				✓	✓	✓	✓	✓
OtW		✓	✓	✓					
ObW		(✓)	(✓)						
OtZ				✓		✓			
Off		(✓)	(✓)	(✓)					
Ofq3		✓	✓	✓		✓			
OpQM				✓		✓			
Opt				✓		✓	✓		
Otp							✓		

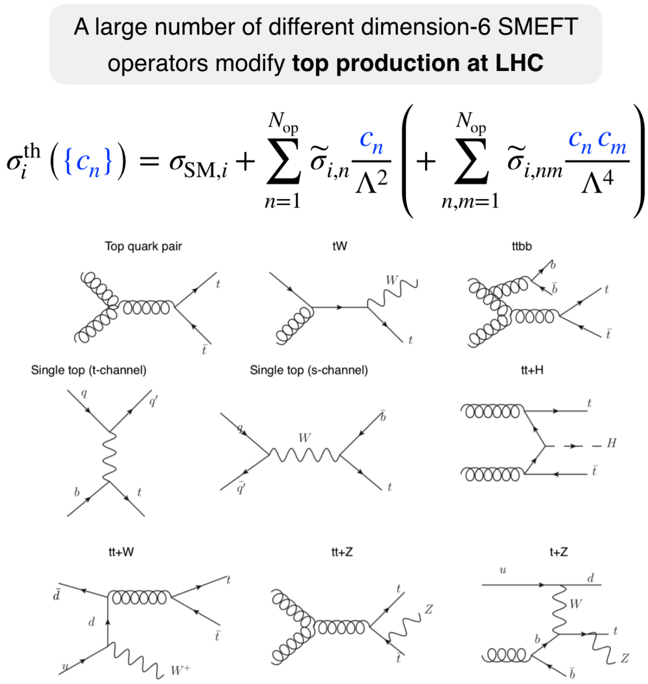


Figure 4.2: In the SMEFT global analysis of top quark data presented in [5], 34 dimension-6 operators (left table) were constrained by 100 cross-sections from around 10 different processes, with the representative Feynman diagrams of each depicted in the right panel.

reasonably large number of operators. A recent example is the global SMEFT analysis of top quark production measurements at the LHC by the `SMEFiT` collaboration presented in [5]. In this case, using the notation introduced in Eq. (4.2), more than $N_{\rm dat} = 100$ independent cross-sections were used to constrain the SMEFT parameter space consisting of $N_{d6} = 34$ operators. A wide range of processes, from inclusive top quark pair and single top production to top quark produced in association with gauge bosons, Higgs bosons, and other heavy quarks was used as input to the analysis.

In the left table of Fig. 4.2 we indicate which of the 34 operators is being constrained by each of the 9 independent processes that was considered in the fit. A tick indicates that this operator is being constrained already at $\mathcal{O}\left(\Lambda^{-2}\right)$, while a tick in parenthesis indicates that these constraints arise only at $\mathcal{O}\left(\Lambda^{-2}\right)$. In the right panel of the same figure we show representative Feynman diagrams corresponding to the 9 different types of processes used in the fit. This figure indicates that only by combining a large number of data from different processes one will be able to constrain all the relevant directions in the SMEFT parameter space without having to impose additional model assumptions.

In order to illustrate how the results of such global SMEFT fit look like, in Fig. 4.3 we display results of the SMEFT analysis of the top quark sector presented in [5]. We show for each of the 34 operators the 95% confidence level lower bounds on the ratio $\Lambda/\sqrt{|c_i|}$. As discussed in Sect. 3, we cannot determine separately the Wilson coefficients from the new physics scale Λ, so only the ratio c_i/Λ^2 is constrained experimentally. For this reason a usual strategy to represent the results of a SMEFT fit is by plotting $\Lambda/\sqrt{|c_i|}$, which corresponds to the indirect reach in energy Λ of the analysis for the case where the associated Wilson coefficient is $\mathcal{O}(1)$. In this case we find that several operators have an energy reach of up to several TeV, which is often larger than the direct reach of the corresponding measurements.

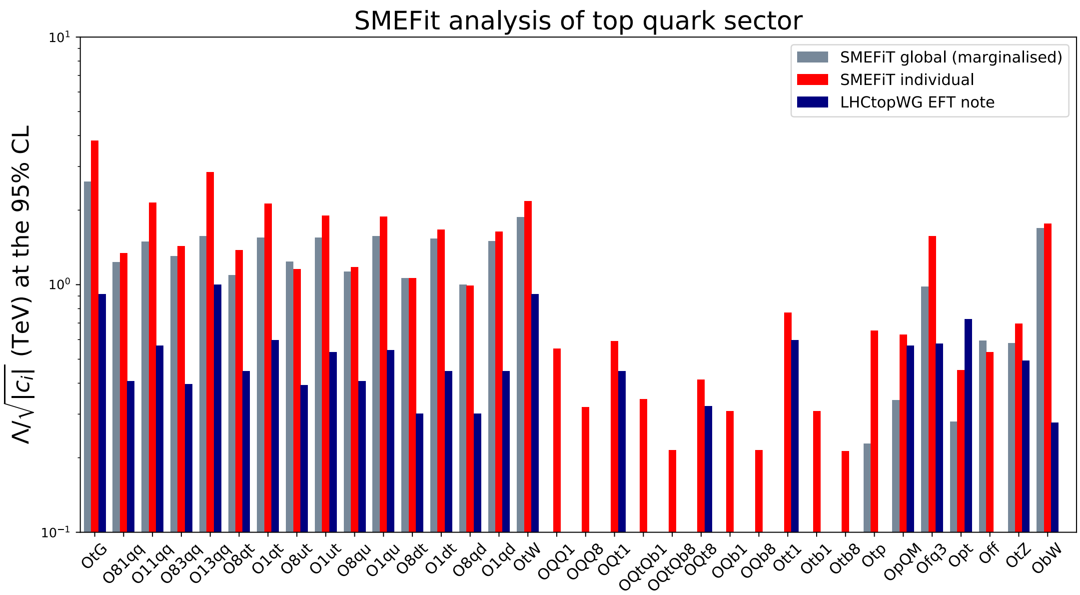


Figure 4.3: The results of the SMEFT analysis of the top quark sector presented in [5]. We show for each of the 34 operators the 95% confidence level lower bounds on the ratio $\Lambda/\sqrt{|c_i|}$.

4.3 Flavour physics

The SMEFT provides a robust framework to correlate different particle sectors. Furthermore, one can construct towers of EFTs starting from the SMEFT itself, for example by integrating heavy SM degrees of freedom which are not relevant at low energies. An important example of this is given by the study of the decays of B-mesons, quark-antiquark bound states that contain one b-quark. As illustrated by Fig. 4.4, in order to describe physics at the scale of the bottom quark, $m_b \simeq 5$ GeV, it is convenient to integrate out the heavy degrees of freedom of the SMEFT: the top quark and the W, Z bosons. The resulting EFT is called the Weak EFT or the Low-Energy EFT. By means of successive applications of the matching procedure, one can relate the Wilson coefficients of the WEFT to those of the SMEFT (plus the masses and couplings of the integrated-out fields) and from there in turn to the masses and couplings of the integrated-out fields of the UV-complete theory.

Fig. 4.4 illustrates an important concept of the EFT framework: while so far we have presented the discussion in terms of a relation between a UV-complete theory and an EFT, in general one can have towers of different EFTs, each of them separated by a mass gap and where the parameters are related among them via subsequent applications of the matching procedure. So in this respect one EFT, such as the SMEFT, can act at the "full" theory of a lower energy EFT, such as the WEFT. The same kind of reasoning works from the high-energy side: it is conceivable that the BSM theory of physics at the few TeV scale could be just an EFT referring to a yet more fundamental theory at higher scales.

The weak EFT

The WEFT is the EFT that one obtains after integrating out the heavy degrees of freedom (W, Z bosons, top quark, Higgs boson) from the SMEFT, and it is only valid for scales $E \lesssim m_b \simeq 5$ GeV. Its operators will be constructed out of the light degrees of freedom of the theory: the u, d, s, c, b quarks, the leptons, and the γ, g gauge bosons.

As usual, these operators will have to satisfy (part of) the symmetries of the SM. The

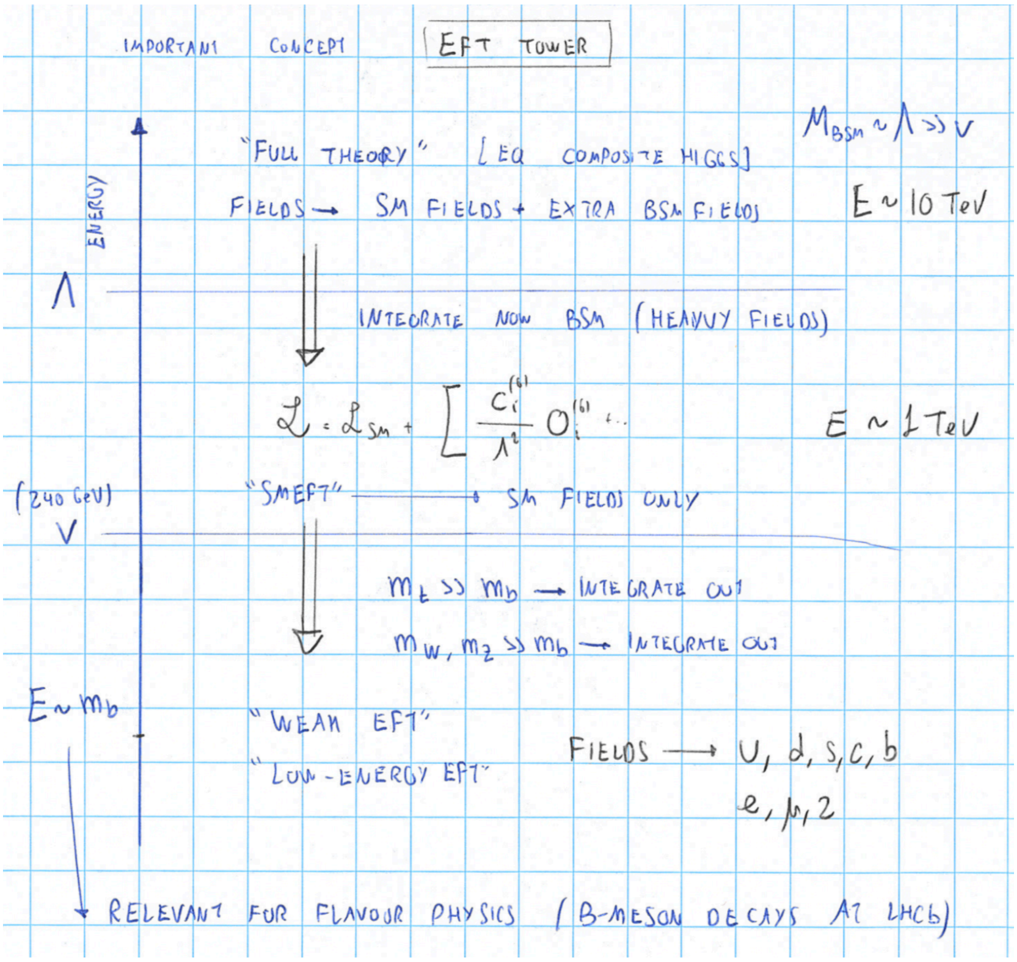


Figure 4.4: In order to describe physics at the scale of the bottom quark, $m_b \simeq 5$ GeV, it is convenient to integrate out the heavy degrees of freedom of the SMEFT: the top quark and the W, Z bosons. The resulting EFT is called the Weak EFT or the Low-Energy EFT.

Wilson coefficients of the WEFT operators will be expressed, upon matching, in terms of the SM couplings, the heavy masses M_W, M_Z, M_t and m_h, and eventually the SMEFT Wilson coefficients as well. Note that at these mass scales the operators will only exhibit gauge invariance under the broken subgroup SU(3)⊗U(1)$_Q$ rather than under the full SM gauge group.

Let us consider a phenomenologically important application of the WEFT. Recently the LHCb experiment has reported deviations with respect to the SM predictions of the rare decay

$$B \to K^* + \mu^+ + \mu^- \,. \tag{4.7}$$

We can compute this decay in the WEFT to correlate with other flavour measurements, and then match to the SMEFT to correlate the LHCb anomalies with other possible effects in high-p_T processes. One of the WEFT operators that are relevant to describe this process is given by

$$\mathcal{O}_9^{bs\mu\mu} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \left(\bar{s}_L \gamma^\rho b_L\right) \left(\bar{\mu} \gamma_\rho \mu\right) \,, \tag{4.8}$$

which is obtained from the SM after integrating out the heavy degrees of freedom. Note that since here the UV complete theory is known (the SMEFT), the top-down matching determines the values of the corresponding Wilson coefficients in terms of known parameters.

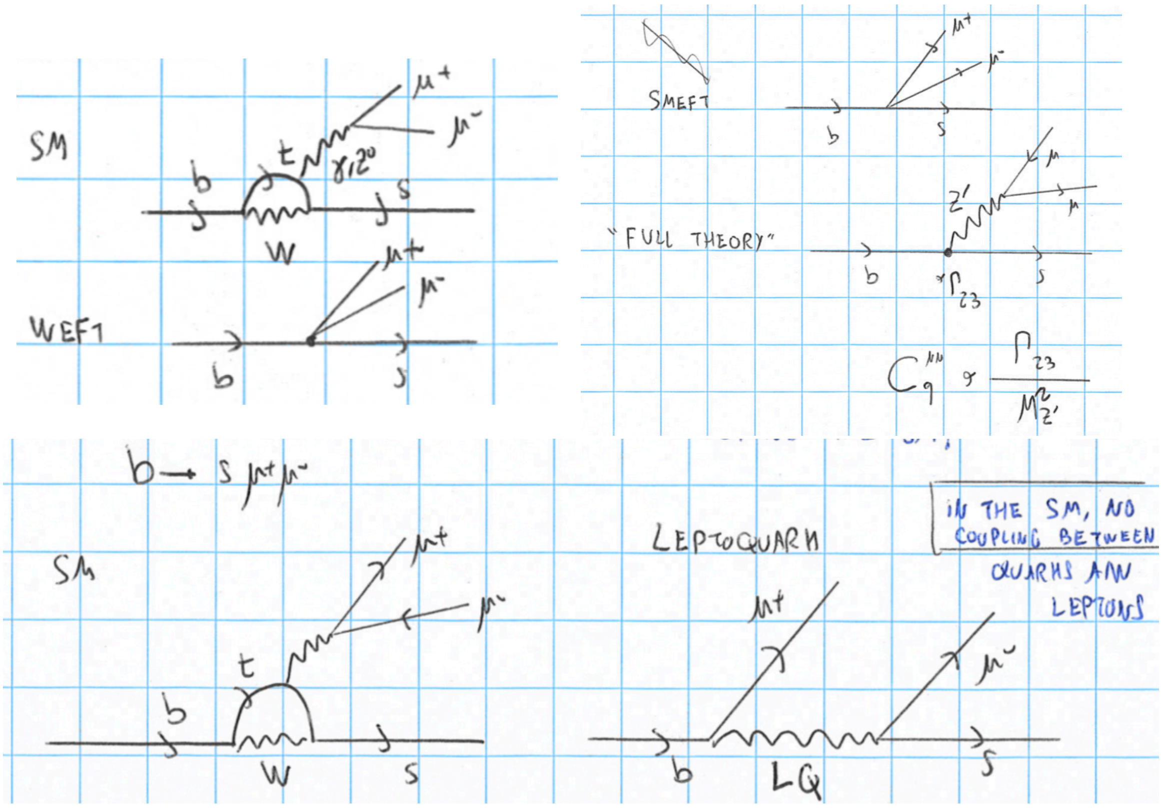


Figure 4.5: The rare B-meson decay $b \to s\mu^+\mu^-$ can be modeled in different ways, as represented by the Feynman diagrams here. In the SM, the process is loop-induced where a γ/Z^0 boson is emitted from the top quark loop. In the Weak EFT, the top quark and the gauge bosons have been integrated out and thus the process takes place via a local four-fermion interaction $b \to s\mu^+\mu^-$. This process can also receive contributions from UV complete models, for example via the emission of a heavy Z' boson or the exchange of a leptoquark (a particle that couples leptons to quarks).

In Fig. 4.5 we represent Feynman diagrams for the various ways in which the rare B-meson decay $b \to s\mu^+\mu^-$ can be modeled. In the SM, the process is loop-induced where a γ/Z^0 boson is emitted from the top quark loop. In the Weak EFT, the top quark and the gauge bosons have been integrated out and thus the process takes place via a local four-fermion interaction $b \to s\mu^+\mu^-$. This process can also receive contributions from UV complete models, for example via the emission of a heavy Z' boson or the exchange of a leptoquark (a particle that couples leptons to quarks). By means of the matching chain UV-complete model $\to$ SMEFT $\to$ WEFT, we can determine how different models affect the WEFT and also what are the implications of these same models for high-p_T processes for which the SMEFT language provides the appropriate mathematical framework.

How we can bridge the connection between WEFT and UV-complete models for this process via the SMEFT? It can be shown that the SMEFT contains four-fermion operators with the suitable field content to contribute to the description of the $b \to s\mu^+\mu^-$ process, such as

$$\mathcal{L}_{\rm SMEFT} \supset \frac{\left[C_{lq}^{(1)}\right]_{2223}}{\Lambda^2}\left[\mathcal{O}_{lq}^{(1)}\right]_{2223}\,, \tag{4.9}$$

where the relevant four-fermion operator is defined as

$$\left[\mathcal{O}_{lq}^{(1)}\right]_{2223} = \left(\bar{l}_2\gamma^\mu l_2\right)\left(\bar{q}_2\gamma_\mu q_3\right) = \left(\bar{\mu}\gamma^\mu\mu\right)\left(\bar{s}\gamma_\mu b\right) + \ldots.... \tag{4.10}$$

where now the Wilson coefficient $\left[C_{lq}^{(1)}\right]_{2223}$ is a free parameter to be extracted from data. Via tree level matching, it is possible to relate the $\mathcal{O}_9^{bs\mu\mu}$ operator in the WEFT with the $\left[\mathcal{O}_{lq}^{(1)}\right]_{2223}$ in the SMEFT. However, here given that these two operators are separated by a large mass gap, it will be very important to appropriately include the effects of operator running and mixing, following the procedure described schematically in Sect. 2.7.

4.4 The SMEFT for Higgs and electroweak physics

As discussed in Sect. 3, several of the dimension-six operators of the SMEFT contain one of more Higgs fields. In general, we can be sensitive to these operators in two ways. Since the Higgs doublet after electroweak symmetry breaking goes to $\varphi \to (0, v+h)/\sqrt{2}$, we can can access these operators via Higgs production at the LHC, as well as to probe them in the vacuum state, where $\langle\varphi\rangle = v$. In particular, these means that we can probe Higgs-related operators via electroweak precision measurements for example at lepton colliders: after all, due to the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetry of the SM the Higgs and electroweak sectors are intimately related.

For example one can consider the following operator

$$\mathcal{O}_L = \left(i\varphi^\dagger D_\mu \varphi\right)\left(\bar{l}_L \gamma^\mu l_L\right) , \tag{4.11}$$

where φ is the Higgs doublet, l_L is a left-handed lepton doublet, and D_μ is the full covariant derivative of the Standard Model, written down for example in Eq. (3.97). Note therefore that via this covariant derivative the Higgs doublet couples to the weak gauge bosons $W^\pm$ and Z. In order to assess to which physical processes this operator will contribute to, we first need to express it once electroweak symmetry has been broken by the Higgs vacuum expectation value. If we keep only the terms proportional to the Higgs vev squared, we see that the SMEFT Lagrangian will contain an interaction of the following form

$$\mathcal{L}_{\rm SMEFT} \supset c_L v^2 Z_\mu \left(\bar{L}_L \gamma^\mu L_L\right) . \tag{4.12}$$

This implies that the operator $\mathcal{O}_L$ will modify the interactions between the Z boson and the leptons, including anomalous couplings to the $Z\bar{e}_L e_L$ and $Z\bar{\nu}\nu$ interaction terms and so forth,

The result of Eq. (4.12) highlights an important property of SMEFT analyses of the Higgs sector: it is not possible to treat separately the Higgs properties from those of the weak gauge bosons, since both sectors are intimately related by electroweak symmetry. So in this respect we can probe the Higgs operators of the SMEFT without the need to actually produce the Higgs itself, but rather by measuring electroweak precision observables such as the properties of the $Z\bar{e}e$ coupling. Indeed, the fact that we can constrain indirectly Higgs physics via precision electroweak measurements is a direct consequence of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry (both in the SM and in the SMEFT), which connects the massive gauge bosons and the Higgs doublet. Experimentally, the possible deformations of these couplings are severely constrained (at the permille level) by the exquisitely precise measurements of Z production in e^+e^- scattering provided by LEP. Therefore any possible deviations from zero of the associated Wilson coefficient c_L are very tightly constrained already from non-Higgs measurements. What happens if we consider only interactions with at least one physical Higgs boson h being produced? Keeping the term linear in h in Eq. (4.11) we find that the analog interaction to Eq. (4.12) will be given by

$$\mathcal{L}_{\rm SMEFT} \supset c_L v h Z_\mu \left(\bar{L}_L \gamma^\mu L_L\right) . \tag{4.13}$$

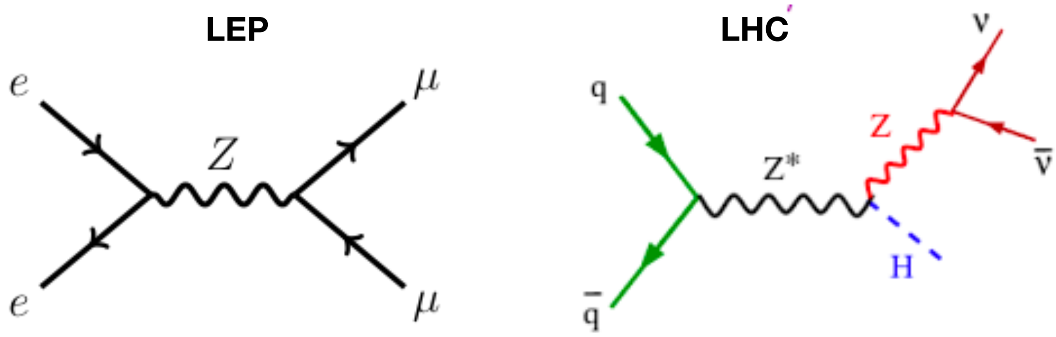


Figure 4.6: The SMEFT operator $\mathcal{O}_L$ defined in Eq. (4.11) can be proved either in electron-positron collisions, via precision electroweak observables, or at the LHC in associated Higgs boson production with a Z boson, as discussed in the text. Note that we can constrain Higgs-related operators in Higgs-less processes via the connection through the Higgs vacuum expectation value.

Now c_L can be constrained in Higgs production processes, for example in associated production where $Z \to l^+l^-$, since then the SMEFT operator will interfere with the SM process. But these bounds are unlikely to be competitive with those arising from the electroweak precision data from LEP. In Fig. 4.6 we represent the SM diagrams related to this case, showing how the SMEFT $\mathcal{O}_L$ defined in Eq. (4.11) can be proved either in electron-positron collisions, via precision electroweak observables, or at the LHC in associated Higgs boson production with a Z boson.

As opposed to those Higgs operators that can already be constrained in the vacuum via electroweak precision observables, for other operators this strategy is not available and therefore we actually need to produce and measure Higgs bosons to access them. In other words, in the absence of the Higgs production measurements from the LHC these SMEFT operators, would be unconstrained. To illustrate when this is the case, let us now consider the operator following dimension-six operator, coupling the Higgs field to two gluon field strength tensors:

$$\mathcal{O}_{GG} = \left(\varphi^\dagger \varphi\right) G^a_{\mu\nu} G^{\mu\nu}_a \,, \tag{4.14}$$

where a is a color index. After electroweak symmetry breaking, the Lagrangian of our theory will now contain the following interactions:

$$\begin{aligned} & \mathcal{L}_{\rm SMEFT} \supset \frac{1}{g_s^2} G_\mu G^{\mu\nu} + \frac{c_{GG}}{\Lambda^2}\left(\varphi^\dagger \varphi\right) G^a_{\mu\nu} G^{\mu\nu}_a \\ \to \quad & \mathcal{L}_{\rm SMEFT} \supset \left(\frac{1}{g_s^2} + \frac{c_{GG} v^2}{\Lambda^2}\right) G^a_{\mu\nu} G^{\mu\nu}_a + \frac{2 v c_{GG}}{\Lambda^2} h G^a_{\mu\nu} G^{\mu\nu}_a \,. \end{aligned} \tag{4.15}$$

Therefore we find that in this case the corrections induced by this operator proportional to v^2 only redefines the strong coupling constant, which anyway needs to be fixed by experiment and thus will not be observable. So the effects of the $\mathcal{O}_{GG}$ operator can only be assessed via the $hG^a_{\mu\nu}G^{\mu\nu}_a$ interaction, which requires the explicit production of a Higgs boson. For example, this term will induce a new interaction proportional to hgg, which will contribute to Higgs production in gluon fusion bypassing the need of the top quark loop. As opposed to the case of the $\mathcal{O}_L$ operator, only by means of Higgs production measurements at the LHC we are able to constrain the value of the corresponding Wilson coefficient c_{GG}.

In a similar way as the top quark case, several SMEFT analyses studying Higgs measurements from the LHC and electroweak precision observables from LEP have been presented, see for example [?, 6]. These analyses include not only Higgs signal strength

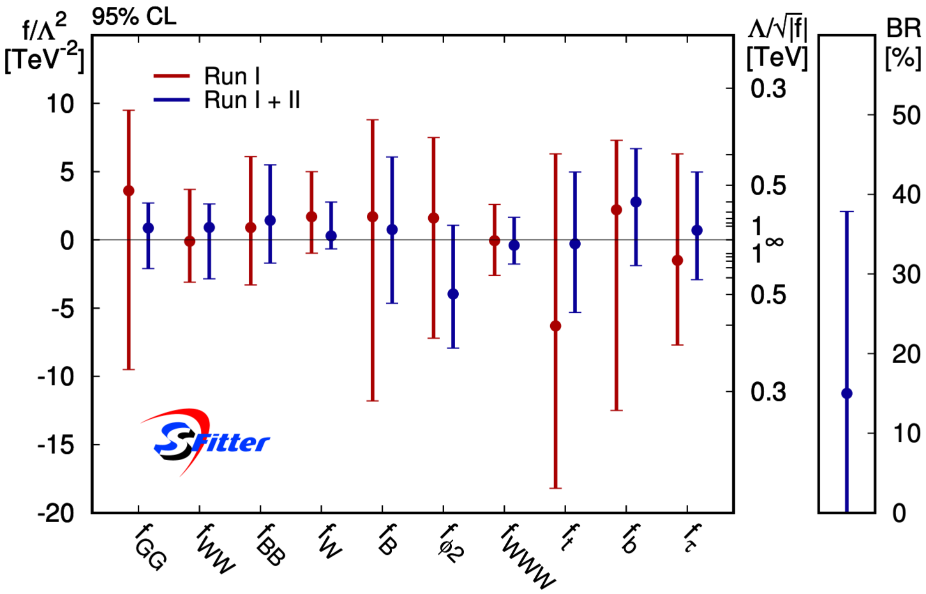


Figure 4.7: The results of the combined SMEFT analysis of Higgs and gauge boson production data from [6].

measurements from the LHC but differential distributions and the so-called simplified template cross-sections. In addition, LHC measurements on gauge boson pair production are also used to complement the information provided by the LEP experiments. In Fig. 4.7 we display the results of the combined SMEFT analysis of Higgs and gauge boson production data from [6]. In the left axis we show the lower bounds on the ratio $\Lambda/\sqrt{|c_i|}$. Note that in this analysis the branching ratio of the Higgs boson to invisible final states (which cannot be determined in a model-independent way at the LHC) is included as an additional free parameter in the fit.

5 Conclusion

In these lectures notes we have aimed to provide a pedagogical introduction to the main features and phenomenological implications of the SMEFT. These lectures are aimed to graduate students with some background in theoretical high energy physics but not necessarily in particle physics: for instance the participants of the DRSTP Theoretical High Energy Physics graduate school were carrying Ph. D. research in topics ranging from quantum gravity to string theory and cosmology. The idea was to start from the basic concepts of effective field theories, such as top-down vs bottom-up constructions, matching at tree and loop level, and operator running and mixing, and apply these ideas to a number of toy models. Then in the second part of the lectures we explained how the SMEFT is constructed and how one can determine the relevant higher-dimensional operators and make predictions for cross-sections and decay widths. We also provided a explicit example of the matching to a UV-complete theory in the SMEFT, for the case where it is extended with a heavy gauge boson with non-universal couplings to quarks. Finally we took a brief look at the SMEFT phenomenology, from the fitting approach to the results in studies of the top, flavour, and top sector.

Needless to say these lecture notes only scrap the surface of the rich physics that is encapsulated by the SMEFT. We have left out many important topics, from subtleties associated to the SMEFT quantisation to a description of the advanced fitting methodologies used in global fits. Likewise, much more could be said on the implications of the SMEFT in top, Higgs, and flavour analysis. Several more technical but definitely important topics have not been considered, such as higher order corrections to SMEFT-induced effects, the automation of the matching between the SMEFT and UV-complete theories, and the limitations on the SMEFT for example in the context of models that contain dark matter candidates or very light particles such as axions. Another topic that has raised a lot of attention is the use of machine learning methods to explore efficiently the large parameter space associated to realistic SMEFT analyses.

The Standard Model Effective Field Theory will be one of the cornerstones of high-energy physics in the medium and long term, with the field bracing up for the long haul of the high-precision program of the high-luminosity LHC. We therefore hope that these lecture notes can provide an useful resource for students aiming to learn a bit more about this important topic.

Acknowledgments

I am grateful to many colleagues for insightful discussions and for scientific collaborations about different aspects of the Standard Model Effective Field Theory: Ilaria Brivio, Fabio Maltoni, Emanuele Nocera, Francesco Riva, Veronica Sanz, Emma Slade, Mike Trott, Eleni Vryodinou, and Cen Zhang.

A Solutions to the tutorial exercises

A.1 Problem I.1

The parity symmetry that the light scalar field ϕ must satisfy imposes that only an even number of scalar fields ϕ can appear in the EFT Lagrangian. Taking into account this consideration as well as other symmetries of the full theory such as Lorentz invariance, the most general EFT for ϕ will be given by

$$\begin{aligned}\mathcal{L}_{\rm eft} &= -\frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \frac{1}{2}m^2\phi^2 + C_{4,0}\phi^4 + C_{6,0}\phi^6 + C_{4,2}\phi^2\left(\partial^\mu\phi\right)^2 \\ &+ C_{8,0}\phi^8 + C_{6,2}\phi^4\left(\partial^\mu\phi\right)^2 + C_{4,4}\left(\partial^\mu\phi\right)^4 \ldots\,,\end{aligned} \tag{A.1}$$

where the dots indicate higher order terms. The coefficients are labelled as $C_{n,m}$ where n the number of fields and m the number of derivatives they contain. The mass dimension of each coefficient is then given by $[C_{n,m}] = 4-(n+m)$. Note that all terms satisfy parity symmetry $\phi\to-\phi$. The terms with derivative coupling can be generated by momentum-dependent terms in the scattering amplitude of the full theory.

In order to carry out the tree-level matching, it is convenient to impose the quality of the 4-point amplitude, $\mathcal{M}\left(\phi\phi\to\phi\phi\right)$. In the full theory this amplitude will be proportional to

$$\mathcal{M}_{\rm full}\left(\phi\phi\to\phi\phi\right) \propto \frac{b^2}{q^2+M^2}\,, \tag{A.2}$$

which can be expanded in Taylor series to give

$$\mathcal{M}_{\rm full}\left(\phi\phi\to\phi\phi\right) \propto \frac{b^2}{M^2} - \frac{b^2}{M^4}q^2 + \ldots\ldots \tag{A.3}$$

with q being the four-momentum carried by the heavy field. Computing the same amplitude in the EFT and matching the results with the full theory allows us to estimate the values of C_4 and $C_{4,2}$,

$$C_4 \propto \frac{b^2}{M^2}\,, \qquad C_{4,2} \propto \frac{b^2}{M^4}\,, \tag{A.4}$$

where we note that derivative couplings in the EFT have associated powers of momentum that match the q^2 contribution in the full theory. Note that $[b]=1$, and therefore the C_4 coupling in the EFT is dimensionless and the $C_{4,2}$ coupling has mass dimensions of -2 as expected from general considerations (see above). The Feynman diagrams for the amplitudes used in the tree-level matching between the full and effective theories are displayed in Fig. A.1 below. Likewise, the matching of the 8-point amplitude between the EFT and the full theory leads to the following relation

$$C_8 \propto \frac{c^2}{M^2}\,, \tag{A.5}$$

whose mass dimensions are $[C_8]=-4$ since $[c]=-1$, respecting the power counting of the EFT expansion. In this case momentum-dependent contributions would only contribute at higher orders in the EFT expansion that we are considering here and can be neglected,

Then one can consider the matching of the six-point amplitude, $\phi\phi\phi\phi\to\phi\phi$. By comparing the diagrams in the full and the effective theories one finds that

$$c_6 \propto \frac{cb}{M^2}\,, \qquad c_{6,2} \propto \frac{cb}{M^4}\,, \tag{A.6}$$

Taking into account these results, Therefore the expression of the interaction term in the EFT Lagrangian after the matching is given by

$$\begin{aligned}
\mathcal{L}_{\rm eft} &= -\frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \frac{1}{2}m^2\phi^2 + a_1\frac{b^2}{M^2}\phi^4 + a_2\frac{cb}{M^2}\phi^6 \\
&+ a_3\frac{b^2}{M^4}\phi^2\left(\partial^\mu\phi\right)^2 + a_4\frac{c^2}{M^2}\phi^8 + a_5\frac{cb}{M^4}\phi^4\left(\partial^\mu\phi\right)^2 + \ldots\,, \qquad \text{(A.7)}
\end{aligned}$$

where a_i are dimensionless coefficients of $\mathcal{O}(1)$ size.

One should also note that the starting theory contains an irrelevant operator, $c\phi^4\Phi$, with negative mass dimension $[c] = -1$. From the discussion in the lectures we know that this operator will not be renormalizable, and therefore the original QFT is also probably an effective rather than a fundamental theory.

The problem also ask us to provide an example of a loop-correction effect that could contribute to the EFT matching. We remark that loop effects are not relevant here. The reason is that for the heavy field Φ to appear in a loop diagram it must couple also with itself via an interaction vertex (else you cannot draw close loops of Φ). Since this is not the case in this theory, loop effects are not relevant for the matching procedure.

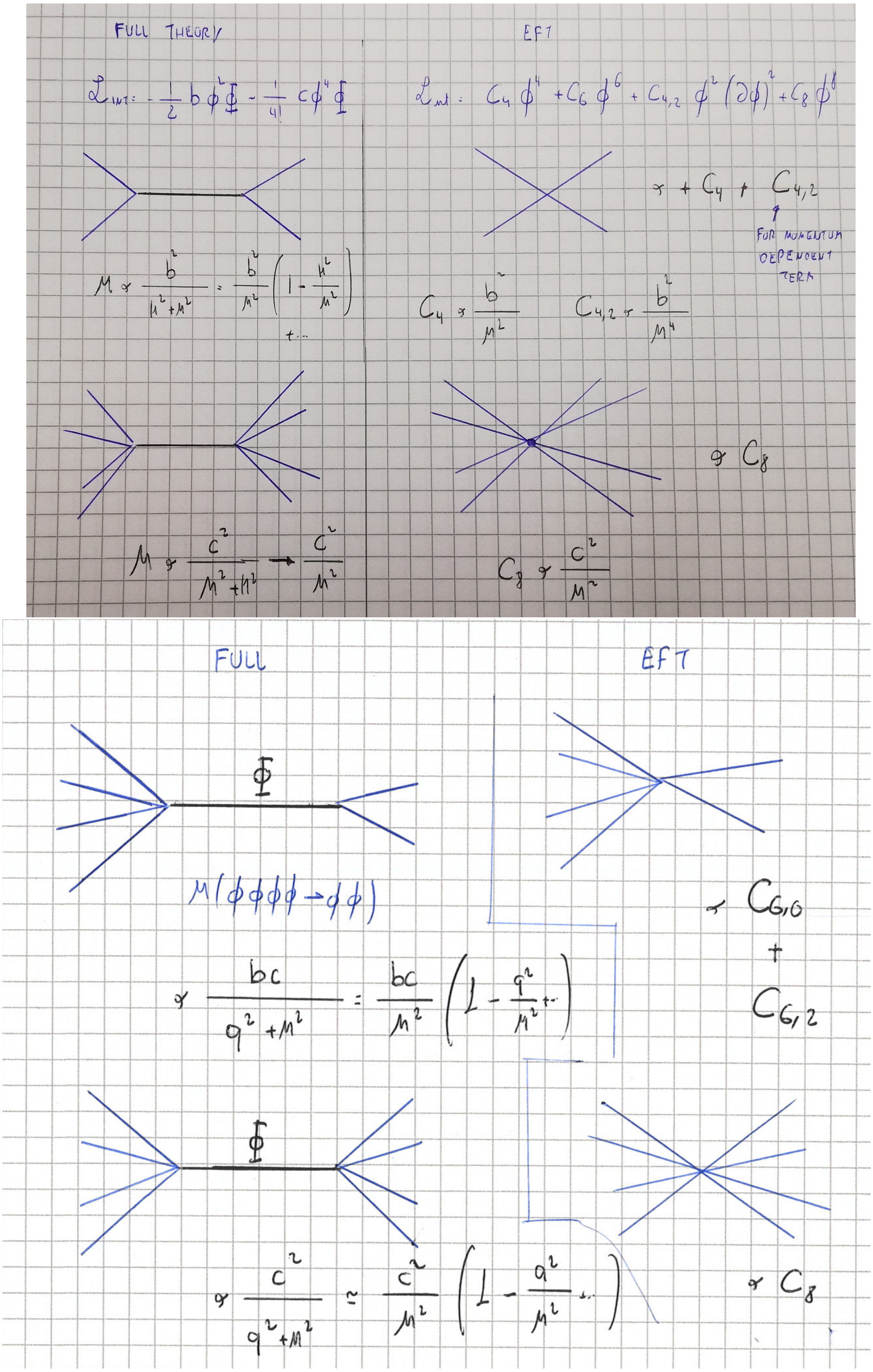


Figure A.1: Diagrams contributing to the tree-level matching between the full and the effective theories in this problem.

A.2 Problem II.1

Let us consider the various cases proposed in this problem for the running of the renormalised coupling constant in turn.

- $n = 0$. In this case the RGE for the running of the coupling reads

$$\frac{dg_R(s)}{d\ln s} = \pm\beta_0\,, \tag{A.8}$$

subject to the boundary condition that $g(s_0) = g_0$. Integrating this equation leads to the result

$$g_R(s) = g_0 \pm \beta_0 \ln\frac{s}{s_0}\,. \tag{A.9}$$

This theory has the interesting feature that for energies such that

$$s = s_0 \exp\left(\mp\frac{g_0}{\beta_0}\right) \tag{A.10}$$

the running coupling vanishes and thus the theory becomes non-interacting (trivial).

Note that in this theory there is no Landau pole, in the sense that for no finite value of s one has a divergence in the value of the running coupling. As mentioned above, in this theory instead of a Landau pole one obtains a trivial fixed point where the coupling becomes exactly zero.

- $n = 1$. In this case the RGE for the running of the coupling reads

$$\frac{dg_R(s)}{d\ln s} = \pm\beta_0 g_R(s)\,, \tag{A.11}$$

subject to the boundary condition that $g(s_0) = g_0$. This differential equation can be integrated to give

$$g_R(s) = g_0\left(\frac{s}{s_0}\right)^{\pm\beta_0} \tag{A.12}$$

The dependence of this coupling with the energy depends on the sign of β_0. For positive β_0 the coupling will grow with the energy polynomially, while for negative β_0 it will decrease with the energy. Note that in this theory there is no Landau pole and the running coupling remains finite for any value of the energy.

- $n = 2$. For this value of the exponent the renormalisation group equations look like:

$$\frac{dg_R(s)}{d\ln s} = \pm\beta_0 g_R^2(s)\,, \tag{A.13}$$

which can be integrated to give, assuming as for the other cases that the coupling is measured to be g_0 at the reference scale s_0,

$$-\frac{1}{g_R(s)} + \frac{1}{g_0} = \pm\beta_0 \ln s/s_0\,, \tag{A.14}$$

which after some rearrangement gives

$$g_R(s) = \frac{g_0}{1 \mp \beta_0 g_0 \ln s/s_0}\,. \tag{A.15}$$

This running coupling has a pole (Landau pole) when the denominator vanishes, and this corresponds to the following value of the energy.

$$s_{\rm L} = s_0 \exp\left(\pm(\beta_0 g_0))^{-1}\right)\,. \tag{A.16}$$

The value of this Landau pole separates the perturbative and non-perturbative regimes of the theory. This kind of running coupling is the one that arises in gauge theories such as in those that constitute the Standard Model.

- Now we consider the final case, where $n = 3$. Here the renormalisation group equations look like:

$$\frac{dg_R(s)}{d\ln s} = \pm\beta_0 g_R^3(s)\,, \tag{A.17}$$

which can be integrated to give, assuming as for the other cases that the coupling is measured to be $g_R(s_0)$ at the reference scale s_0,

$$-\frac{1}{2g_R^2(s)} + \frac{1}{2g_0^2} = \pm\beta_0 \ln s/s_0\,, \tag{A.18}$$

which after some rearrangement gives

$$g_R(s) = \frac{g_0}{\left(1 \mp \beta_0 g_0^2 \ln s/s_0\right)^{1/2}}\,. \tag{A.19}$$

This running coupling has a pole (Landau pole) when the denominator vanishes, and this corresponds to the following value of the energy

$$s_{\rm L} = s_0 \exp\left(\pm(\beta_0 g_R^2(s_0))^{-1}\right)\,. \tag{A.20}$$

The value of this Landau pole separates the perturbative and non-perturbative regimes of the theory. Note also that the qualitative behaviour of $g_R(s)$ is quite similar for the cases in which $n = 2$ and $n = 3$, which in turn is quite different from those in the other two cases.

You can check that it is possible to integrate the general RGE

$$\frac{dg_R(s)}{d\ln s} = \pm\beta_0 g_R^n(s)\,, \tag{A.21}$$

for any other value $n \leq 3$. In all cases we will find a Landau pole, unlike the theories defined by the RGEs with $n = 0$ and $n = 1$ for which such a Landau pole was absent.

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